Tsukuba Conference on Integral Geometry and Harmonic Analysis

Date: August 7 (Monday) – August 10 (Thursday), 2006

Place: Laboratory of Advanced Research B, University of Tsukuba
1-1-1, Tennodai, Tsukuba-shi, Ibaraki, 305-8577, Japan (cf. WEB site at the bottom)

PROGRAM

August 7 (Monday)

13:00-13:10 Opening remarks

Chairperson: Toshio Oshima (University of Tokyo)

13:10-14:10 Sigurdur Helgason (Massachusetts Institute of Technology)
The X-ray Transform
Abstract: In this lecture we discuss the X ray transform on a symmetric space, compact and non compact. For the noncompact case the theory is fairly complete, injectivity, support theorems and inversion formulas. We will discuss recent progress in the area.

14:10-14:30 *Break*

14:30-15:30 Satoshi Ishikawa (University of Tokyo)
The Radon Transform for the double fibrations of noncompact symmetric spaces
Abstract: I present the results on the operational properties of the Radon Transform for the double fibrations of noncompact symmetric spaces associated with the inclusion incidence relations. Our approach is based on the projection slice theorem which relates the Radon transform and the Fourier transform on symmetric spaces and the underlying geometry of the double fibrations also plays an important role.

15:30-15:50 *Break*

15:50-16:50 Yutaka Matsui (University of Tokyo)
Topological Radon transforms and their applications (joint with Kiyoshi Takeuchi)
Abstract : Topological Radon transforms of (subanalytically) constructible functions are defined by taking the topological Euler characteristics of hyperplane sections of subanalytic sets. We study topological Radon transforms of constructible functions on projective spaces or Grassmann manifolds by means of the combinatorial Schubert calculus and the microlocal theory of sheaves developed by Kashiwara-Schapira. In particular, we introduce inversion formulas of them and the characterization of their images. Some applications to algebraic geometry will be also given.

August 8 (Tuesday)

Chairperson: Gestur Olafsson (Louisiana State University)

10:00-11:00 Hiroshi Isozaki (University of Tsukuba)
Inverse boundary value problems and hyperbolic spaces
Abstract: We present a new approach to the inverse problem based on the structure of hyperbolic space. There are 3 main issues :

- Solving the inverse boundary value problems in $\mathbf{R}^n (n \geq 3)$ by using hyperbolic manifolds
- Inverse boundary value problems in the horosphere
- Detection of location of inclusions from the local DN map

The starting point is the following observation. Given a boundary value problem $(-\Delta+q)u = 0$ in $\Omega \subset \mathbf{R}^n$, let $v = x_n^{(2-n)/2}u$. Then v satisfies $(-\Delta_g+V)v = 0$, where $V = x_n^2 q - n(n-2)/4$ and $\Delta_g = x_n^2\partial_n^2 - (n-2)x_n\partial_n + x_n^2\Delta_x$, which is just the Laplace-Beltrami operator on $\mathbf{H}^n$. (We denote the points in $\mathbf{R}^n$ by $(x, x_n), x \in \mathbf{R}^{n-1}$, and the domain Ω is regarded to sit in the region $\{x_n > 0\}$). This means that the inverse boundary value problem in $\mathbf{R}^n$ and the one in $\mathbf{H}^n$ are equivalent. To study the latter, the main tool we use is the Green function of the gauge-transformed operator $H_0(\theta) = -e^{-ix\cdot\theta}\Delta_g e^{ix\cdot\theta}, \theta \in \mathbf{C}^{n-1}$, which is written in terms of modified Bessel functions : $e^{i(x-x')\cdot\xi}(yy')^{n-1)/2}I_\nu(\zeta(\xi,\theta)y)K_\nu(\zeta(\xi,\theta)y), \zeta(\xi,\theta) = \sqrt{(\xi+\theta)^2}$. This is a counter part of Faddeev's Green function $\int e^{i(x-x')\cdot\xi}(\xi^2+2\zeta\cdot\xi)^{-1}d\xi$, a fundamental tool in the inverse scattering theory for Schrödinger operators on the Euclidean space.

(1) Inverse boundary value problems. To solve the inverse boundary value problem in $\mathbf{H}^n$, we pass it to the quotient space $\mathbf{H}^n/\Gamma$, where Γ is a lattice in $\mathbf{R}^{n-1}$ of rank $n-1$ which is taken so large that the fundamental domain $E = \mathbf{R}^{n-1}/\Gamma$ contains the region Ω completely inside. In this case, θ varies over the fundamental domian of the dual lattice of Γ and is regarded as a Floquet parameter in the theory of periodic Schrödinger operators. From the Dirichlet-Neumann map for the boundary value problem, one can construct the scattering amplitude. This latter is analytic in some region of complexified Floquet parameters θ. By passing to this parameter to infinity (complex Born approximation), one can reconstruct q from the DN map.

(2) Horosphere boundary value problem and Barber-Brown algorithm. Consider a boundary value problem for the Schrödinger operator $-\Delta+q(x)$ in a ball $\Omega : (x_1+R)^2+x_2^2+(x_3-r)^2 < r^2$, whose boundary we regard as a horosphere in the hyperbolic space $\mathbf{H}^3$ realized in the upper half space. Let $S = \{|x| = R, x_3 > 0\}$ be a hemisphere, which is generated by a family of geodesics in $\mathbf{H}^3$. By imposing a suitable boundary condition on $\partial\Omega$ in terms of a pseudo-differential operator, we compute the integral mean of $q(x)$ over $S \cap \Omega$ from the associated (generalized) Robin-to-Dirichlet map for $-\Delta + q(x)$. The potential $q(x)$ is then reconstructed by virtue of the inverse Radon transform on hyperbolic space. This justifies the well-known Barber-Brown algorithm in electrical impedance tomography.

(3) Detection of inclusions - Applications to numerical computation. This hyperbolic space approach can also be used to detect the location of non-smooth part of conductivity $\gamma(x)$ of a body Ω in $\mathbf{R}^d$, $d = 2,3$. Suppose for the sake of simplicity that we know the DN map Λ_0 for the case that γ is a constant, and that the conductivity is different from this constant on a subset $\Omega_1 \subset \Omega$. Take x_0 from outsied of the convex hull of Ω and let $S^\epsilon_{out} = \{x \in \partial\Omega; |x-x_0| > R+\epsilon\}, S^\epsilon_{in} = \{x \in \partial\Omega; |x-x_0| < R-\epsilon\}$. Then one can construct the boundary data $f_\tau(x)$ depending on a large paramter $\tau > 0$ having the following properties : On S^ϵ_{out} (S^ϵ_{in}), $f_\tau(x)$ is exponentially decreasing (increasing) in τ. Let Λ be the DN map for γ. If $R < dis(x_0,\Omega_1)$, then $0 \leq (\Lambda-\Lambda_0)f_\tau, f_\tau) < Ce^{-\delta\tau}$, and if $R > dis(x_0,\partial\Omega_1)$, then $((\Lambda-\Lambda_0)f_\tau, f_\tau) > C'e^{\delta\tau}$. This means that one can detect the location of inclusions from the boundary data which are essentially localized on a part of the boundary.

References

[Is1] H. Isozaki, *Inverse spectral problems on hyperbolic manifolds and their applications to inverse boundary value problems in Euclidean space*, Amer. J. Math. 126 (2004), 1261-1313.

[Is2] H. Isozaki, *Inverse boundary value problems in the horosphere - A link between hyperbolic geometry and electrical impedance tomography*, preprint.

[IINSU] T. Ide, H. Isozaki, S. Nakata, S. Siltanen and G. Uhlmann, Probing for electrical inclusions with complex spherical waves, to appeai in C.P.A.M..

11:00-11:20 *Break (group picture)*

11:20-12:20 Eric Todd Quinto (Tufts University)

Support Theorems for the Spherical Radon Transform on Manifolds

Abstract: Let M be a real-analytic manifold and let S be a real-analytic hypersurface. We prove a local support theorem for the spherical Radon transform that integrates over spheres

centered at points on S. Our theorem is valid for distributions supported on one side of "tangent surfaces" to S. The proof involves the microlocal analysis of the sphere transform and a microlocal Holmgren theorem of Kawai, Kashiwara, and Hoermander. Generalizations and possible applications to sonar and the wave equation will be outlined.

12:20-14:30 *Lunch Time*

Chairperson: Eric Grinberg (University of New Hampshire)

14:30-15:30 Boris Rubin (Louisiana State University)
Integral geometry and fractional calculus
Abstract: Fractional calculus is a branch of analysis which studies different kinds of fractional integrals, their modifications and generalizations. These include Riemann-Liouville integrals, Riesz potentials, analytic families of cosine transforms, and many others. Diverse (well known and new) fractional integrals arise in a natural way in integral geometry. I am planning to talk about application of fractional calculus to Radon transforms of different types and the Busemann-Petty problem on sections of convex bodies.

15:30-15:50 *Break*

15:50-16:50 Takaaki Nomura (Kyushu University)
Tube domain and an orbit of a complex triangular group (joint with Hideyuki Ishi)
Abstract: Let w be a complex symmetric matrix of order r, and $\Delta_1(w), \ldots, \Delta_r(w)$ the principal minors of w. If w belongs to the Siegel right half space, then we have

$$\operatorname{Re} \frac{\Delta_k(w)}{\Delta_{k-1}(w)} > 0 \quad (k = 1, \ldots, r). \qquad (*)$$

The property $(*)$ is true for general symmetric right half spaces if we consider Δ_k in the framework of Jordan algebras. Now principal minors are basic relative invariants for the triangular group, and in this way we can consider $(*)$ even in the nonsymmetric case. For nonsymmetric right half spaces, the above property $(*)$ is in general false. But we will present a series of nonsymmetric right half spaces with the property $(*)$. The proof of $(*)$ for symmetric spaces consists of two parts. One part is generalized to nonsymmetric cases, while the other is generally false there. This will be the final part of the talk.

18:00-20:00 *Reception (Tremont Hotel)*

August 9 (Wednesday)

Chairperson: Fulton Gonzalez (Tufts University)

10:00-11:00 Toshiyuki Kobayashi (Kyoto University)
Conformally invariant Hilbert structure on the solution space of the ultrahyperbolic Laplace equation on $\mathbb{R}^{p+q}$
Abstract: The indefinite orthogonal group $O(p+1, q+1)$ acts as the Möbius group of conformal transformations on $\mathbb{R}^{p+q}$, and preserves the space of solutions of the ultrahyperbolic Laplace equation on $\mathbb{R}^{p+q}$. Inspired by the idea of Sato's hyperfunctions, we construct in an intrinsic and natural way a Hilbert space of solutions by the integration of the data of solutions over a hypersurface, and give a new construction of its minimal unitary representation for $p + q$ even.

11:00-11:20 *Break*

11:20-12:20 Eric Grinberg (University of New Hampshire)
TBA

12:20-14:30 *Lunch Time*

Chairperson: Toshiyuki Kobayashi (Kyoto University)

14:30-15:30 Fulton Gonzalez (Tufts University)
The range of the matrix Radon transform (joint with Tomoyuki Kakehi)
Abstract: In this talk I will describe a range characterization of the matrix Radon transform by invariant differential operators, in the case when the source manifold (in this case $M_{n,k}$, the vector space of $n \times k$ real matrices) has lower dimension than the target manifold, the space Ξ of affine matrix planes. This generalizes analogous results, given by the group-invariant form of John's equations, characterizing the range of the d-plane transform in $\mathbb{R}^n$.

15:30-15:50 *Break*

15:50-16:50 Gen Mano (Kyoto University)
The unitary inversion operator for the minimal representation of O(p,q)
Abstract: The L^2-model of the minimal representation of the indefinite orthogonal group $O(p,q)$ ($p+q$: even, greater than 4) was established by Kobayashi-Orsted (2003). In this talk, we present an explicit formula for the unitary inversion operator, which plays a key role in this L^2-model. Our proof uses the analysis on the Radon transform of distributions supported on the light cone.

August 10 (Thursday)

Chairperson: Sigurdur Helgason (Massachusetts Institute of Technology)

9:00-10:00 Hideko Sekiguchi (University of Tokyo)
Radon-Penrose transform for the quantization of elliptic orbits
Abstract: The Radon-Penrose transform is defined on the quantization of elliptic coadjoint orbits as intertwining operators of the transformation group. In this talk, I will give the characterization of the Penrose transform in the degenerate case for real symplectic groups.

10:00-10:20 *Break*

10:20-11:20 Gestur Olafsson (Louisiana State University)
Holomorphic Aspects of the Abel Transform

11:20-11:40 *Break*

11:40-12:40 Toshio Oshima (University of Tokyo)
Differential equations attached to generalized flag manifolds
Abstract: Generalized flag manifolds are homogeneous spaces of real reductive Lie groups where the degenerate principal series representations of the groups. We give explicit expressions of the annihilators of the representations and then the images of equivariant integral transformations, such as Poisson, Radon and Penrose transformations etc., satisfy differential equations corresponding to the annihilators.

For the details of the conference, please visit the following site:
http://www.math.tsukuba.ac.jp/~kakehi/integral-geometry/conference.html

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