18.784 Homework Set 2
Due Friday, February 12, 2010.

Part I (QY, 2/8/10)

1. Show that $SL_2(\mathbb{Z})$ is generated by $S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. (Hint: Use the Euclidean algorithm)
2. Show that the only fractional linear transformations $z \mapsto \frac{az+b}{cz+d}$ which preserve $\mathbb{H}$ are those with $ad - bc = 1$ and $a, b, c, d \in \mathbb{R}$.

Part II (MD, 2/10/10)

1. Let $\frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$. Show that if a function f satisfies the Cauchy-Riemann equations, then $\frac{\partial f}{\partial \bar{z}} = 0$.
2. Let $f(z) = y - x - 3ix^2$. Integrate $f(z)dz$ along the following contours:
   (a) C_1, which is a straight path from 0 to i, followed by a straight path from i to $1 + i$.
   (b) C_2, which is a straight path from 0 to $1 + i$.