Ver. 2017 年 5 月 22 日

Algebraic Lie Theory and Representation Theory (ALTReT) 2017

2017 年 6 月 10 日 (土) ～ 6 月 13 日 (火)
於：帝人アカデミー富士 (静岡県裾野市下和田 656)

この研究集会は，基盤研究 (A) 17H01086 (研究代表者：荒川 知幸) および
基盤研究 (B) 16H03920 (研究代表者：内藤聡) からの支援を受けています．

世話人：内藤 聡 (東京工業大学) naito AT math.titech.ac.jp
池田 岳 (岡山理科大学) ike AT xmath.ous.ac.jp
協力：佐垣 大輔 (筑波大学) sagaki AT math.tsukuba.ac.jp

6 月 10 日 (土)

14:00–15:00 Registration

15:00–16:30 疋田 辰之 (Hikita Tatsuyuki) 京都大学
Canonical bases in equivariant K-theory of conical symplectic resolutions

16:40–17:30 元良 直輝 (Genra Naoki) 京都大学数理研
Wakimoto representations of W-algebras

17:40–18:30 佐藤 僚 (Sato Ryo) 東京大学
Modular invariant representations of the $N = 2$ superconformal algebra

6 月 11 日 (日)

09:00–09:50 藤田 遼 (Fujita Ryo) 京都大学
Tilting modules in affine highest weight categories

10:00–11:00 加藤 周 (Kato Syu) 京都大学
Geometry of the semi-infinite flag manifold and representation theory (part 1)

11:10–12:00 渡邉 英也 (Watanabe Hideya) 東京工業大学
Crystal basis theory for a quantum symmetric pair

— Lunch —

13:30–14:30 入谷 寛 (Iritani Hiroshi) 京都大学

Givental quantization and Fock sheaf (part 1)

14:40–15:30 井上 玲 (Inoue Rei) 千葉大学

Cluster algebraic structure of geometric R-matrices

— Tea Break —

16:00–16:50 大川 領 (Ohkawa Ryo) 早稲田大学

Instanton counting on the minimal resolution of the A_1-singularity

17:00–17:50 柳田 伸太郎 (Yanagida Shintarou) 名古屋大学

Torus skein algebra and mirror symmetry

— Banquet —

6 月 12 日 (月)

09:00–09:50 廣嶋 透也 (Hiroshima Toya) 大阪大学

Crystal interpretation of a formula on the branching rule of C_n type

10:00–11:00 加藤 周 (Kato Syu) 京都大学

Geometry of the semi-infinite flag manifold and representation theory (part 2)

11:10–12:00 八尋 耕平 (Yahiro Kohei) 東京大学

Integral transforms of D-modules on G-varieties

— Lunch —

13:30–14:30 入谷 寛 (Iritani Hiroshi) 京都大学

Givental quantization and Fock sheaf (part 2)

14:40–15:30 柴田 大樹 (Shibata Taiki) 岡山理科大学

Affine Kac-Moody groups as central extensions of loop groups

— Tea Break —

16:00–16:50 中筋 麻貴 (Nakasuji Maki) 上智大学

Schur type multiple zeta functions and its determinant formulae

17:00–17:50 木村 嘉之 (Kimura Yoshiyuki) 大阪府立大学

Twist automorphisms on quantum unipotent cells and the dual canonical bases

— Dinner —

19:45–21:00 ポスターセッション

- 河野 隆史 (Kouno Takafumi) 東京工業大学
  Decomposition of the tensor product of Demazure crystals
- 中山 勇祐 (Nakayama Yusuke) 岡山理科大学 &
  西口　明宏 (Nishiguchi Akihiro) 岡山理科大学
  Review on Riemann-Roch's theorem
- 川合遼太郎 (Kawagou Ryoutarou) 岡山理科大学
  Resolutions of determinantal ideals
- 越田真史 (Koshida Shinji) 東京大学
  Bosonization of affine Lie algebras and integrable hierarchies
- 櫻井太朗 (Sakurai Taro) 千葉大学
  Central elements of the Jennings basis and certain Morita invariants

6 月 13 日 (火)

9:00–9:50 源嶋 孝太 (Gejima Kohta) 大阪大学

An explicit formula of the unramified Shintani functions on $GSp(4)$

10:00–10:50 大矢 浩徳 (Oya Hironori) 東京大学

Twist automorphisms on quantum unipotent cells and the Chamber Ansatz

11:00–11:50 桑原 敏郎 (Kuwabara Toshiro) 筑波大学

Vertex algebras associated with hypertoric varieties

アブストラクト

井上 玲 (Inoue Rei) 千葉大学

Cluster algebraic structure of geometric R-matrices

The geometric R-matrices are 'rational' version of the combinatorial R-matrices acting on crystals. In this talk we introduce cluster R-matrices as sequences of mutations in triangular grid quivers on a cylinder, and show that the affine geometric R-matrix of symmetric tensor representations is obtained from the cluster R-matrix. Further we study a quantization of the affine geometric R-matrix, compatible with the quantum cluster structure a la Fock and Goncharov. We also mention quantum analogues of the loop symmetric functions, which are the invariants of the quantum affine geometric R-matrix.

This work is based on a joint work with Thomas Lam and Pavlo Pylyavskyy.

大川 領 (Ohkawa Ryo) 早稲田大学

Instanton counting on the minimal resolution of the A_1-singularity

I introduce Nekrasov partition functions defiend from A_1-singularity. These are generating functions of integrations over moduli of framed sheaves on resolutions of A_1-singularity. We consider two resolutions, the minimal resolution and a stacky resolution, that is, the quotient stack of the projective plane by the cyclic group of order 2. I will show functional equations between Nekrasov partition functions defined from these two resolutions.

大矢 浩徳 (Oya Hironori) 東京大学

Twist automorphisms on quantum unipotent cells and the Chamber Ansatz

Berenstein, Fomin and Zelevinsky introduced twist automorphisms on unipotent cells in order to describe the inverses of certain embeddings of tori into unipotent cells. The resulting descriptions are called the Chamber Ansatz formulae.

In this talk, we provide quantum analogues of the Chamber Ansatz formulae. Indeed, quantum analogues of twist automorphisms on unipotent cells have been constructed by Kimura and the speaker in general settings. Roughly speaking, we give the explicit description of “the inverse of a certain embedding of a quantum torus into the quantum unipotent cell”, though we do not have the actual spaces but have their coordinate rings. This proves that our quantum twist automorphisms are the generalization of Berenstein-Rupel's ones on unipotent cells associated with the squares of acyclic Coxeter elements.

By the way, there also exist many embeddings of the quantum tori into the quantum unipotent cells which come from the quantum cluster algebra structures. Hence we next discuss the compatibility between the quantum twist automorphisms and the quantum cluster algebra structures.

A part of this talk is based on a joint work with Yoshiyuki Kimura.

木村 嘉之 (Kimura Yoshiyuki) 大阪府立大学

Twist automorphisms on quantum unipotent cells and the dual canonical bases

Let G be a connected simply-connected complex simple algebraic group with a fixed maximal torus H, a pair of Borel subgroups $B_\pm$ such that $B_+ \cap B_- = H$ and a Weyl group $W = \mathrm{Norm}_G(H)/H$ and the maximal unipotent subgroups $N_\pm \subset B_\pm$. For a Weyl group element $w \in W$, we consider the unipotent cell $N_- \cap B_+ \dot{w} B_+$, where $\dot{w}$ is a lift of w in $\mathrm{Norm}_G(H)$. Berenstein, Fomin and Zelevinsky introduced certain automorphism on the unipotent cell, called twist automorphism, for solving “the factorization problems” which describes the inverse of “toric chart” of the associated Schubert varieties.

The quantum unipotent cell is a quantum analogue of the coordinate ring of $N_- \cap B_+ \dot{w} B_+$ which was introduced by De Concini and Procesi and they proved an isomorphism between it and a quantum analogue of the coordinate ring of $N_-(w) \cap \dot{w} G_0$, where $N_-(w) = N_- \cap \dot{w} N_+ \dot{w}^{-1}$ and $\dot{w} G_0$ is the “Gauss” cell associated with w.

In this talk, we construct a quantum analogue of the twist automorphism, called a quantum twist automorphism, as a composite of the De Concini-Procesi isomorphism and the twist isomorphism between $N_- \cap B_+ \dot{w} B_+$ and $N_-(w) \cap \dot{w} G_0$ which is defined by Gaussian decomposition and study its basic properties. In fact, we proved that the the quantum twist automorphism preserves the dual canonical basis of the quantum unipotent cell. This is a joint work with Hironori Oya.

桑原 敏郎 (**Kuwabara Toshiro**) 筑波大学

Vertex algebras associated with hypertoric varieties

We construct a family of vertex algebras associated with a family of symplectic singularity/resolution, called hypertoric varieties. While the hypertoric varieties are constructed by a certain Hamiltonian reduction associated with a torus action, Our vertex algebras are constructed by (semi-infinite) BRST reduction. The construction works algebro-geometrically and we construct sheaves of $\hbar$-adic vertex algebras over hypertoric varieties which localize the vertex algebras. We show when the vertex algebras are vertex operator algebras by giving explicit conformal vectors. We also show that the Zhu algebras of the vertex algebras, associative algebras associated with non-negatively graded vertex algebras, coincide with the universal families of filtered quantizations of the coordinate rings of the hypertoric varieties.

源嶋 孝太 (**Gejima Kohta**) 大阪大学

An explicit formula of the unramified Shintani functions on $GSp(4)$

Let F be a non-archimedean local field of *arbitrary* characteristic. We give an explicit formula of the unramified Shintani functions on $\mathbf{GSp}_4(F)$. As an application, we evaluate a local zeta integral of Murase–Sugano type, which turns out to be the spin L-factor of $\mathbf{GSp}_4$.

元良 直輝 (**Genra Naoki**) 京都大学数理研

Wakimoto representations of W-algebras

(アファイン)W 代数と呼ばれる二次元共形場理論に端を発する, 重要な頂点代数のクラスがある. W 代数は Lie 代数 $\mathfrak{g}$ とそのべき零元 f, 複素数 k に付随した Drinfeld-Sokolov 還元による BRST コホモロジーによって定義される. $\mathfrak{g}=\mathfrak{sl}_2$ の場合に W 代数は Virasoro 代数になる, という意味で W 代数は Virasoro 代数の自然な一般化である. Virasoro 代数の場合には, Heisenberg 代数の Fock 表現を考えることで, Virasoro 代数の自由場実現を介して Fock 空間に作用させることができ, その既約性の判定や特異ベクトルの特定を, intertwining operator を用いて調べることが出来た (Tsuchiya-Kanie など). このような応用を考えて W 代数の自由場実現を構成したい. f が正則べき零元の場合には, generic な k に対して, Feigin-Frenkel によって screening operator を用いて構成されている. 本講演では, アファイン頂点代数の自由場実現である脇本表現を用いて, 一般の W 代数の

自由場実現を (generic な k に対して) 構成する．このとき (一般の)W 代数は, screening operator たちの核として, Heisenberg 頂点代数といくつかの $\beta\gamma$ システムのテンソル積の部分頂点代数として構成される．

佐藤 僚 (Sato Ryo) 東京大学

Modular invariant representations of the $N = 2$ superconformal algebra

One of the most remarkable features in the representation theory of vertex operator superalgebras (VOSAs) is the modular invariance property of the characters of their modules.

In this talk we provide new modular invariant families of (finitely or uncountably many) modules over the simple $N = 2$ vertex operator superalgebra of central charge $3\left(1 - \frac{2p'}{p}\right)$, where (p, p') is a pair of coprime positive integers such that $p \geq 2$. When $p' = 1$, our result coincides with the modular invariance property obtained by F. Ravanini and S.-K. Yang. In addition, we give a remark on the conjectural relationship between the "modular S-matrix" and fusion rules.

柴田 大樹 (Shibata Taiki) 岡山理科大学

Affine Kac-Moody groups as central extensions of loop groups

Untwisted/twisted affine Lie algebras are well understood and have a lot of applications not only in mathematics but also in theoretical physics. On the other hand, infinite-dimensional "Lie groups" constructed from given affine Lie algebras (á la C. Chevalley), which we shall call affine Kac-Moody groups, seems to be less understood. D. Peterson and V. Kac (1983) mentioned and Y. Chen (1996) proved that an untwisted affine Kac-Moody group can be realized as a central extension of an "algebraic loop group". At the affine Lie algebra level, this phenomenon is well-known.

In this talk, we generalize the result to all twisted cases. Namely, we see that a similar central extension result holds for all twisted affine Kac-Moody groups, by using the notions of twisted Chevalley groups over a commutative ring (defined by E. Abe, 1976). As an application, we show that a twisted affine Kac-Moody group is isomorphic to a certain fixed-points subgroup of an untwisted Kac-Moody group. The result seems to be closely related to Galois descent theory for affine Lie algebras. This is a joint work with J. Morita (University of Tsukuba) and A. Pianzola (University of Alberta).

中筋 麻貴 (Nakasuji Maki) 上智大学

Schur type multiple zeta functions and its determinant formulae

To interpolate both multiple zeta and multiple zeta-star functions of Euler-Zagier type, we introduce a new family of multivariable functions, called Schur multiple zeta functions, defined combinatorially in terms of semi-standard Young tableaux as an analogue of classical Schur functions. We establish some determinant formulae coming from theory of the original Schur functions such as Jacobi-Trudi, Giambelli, and dual Cauchy formula. These formulae lead to quite non-trivial algebraic relations among multiple zeta and zeta-star functions. We also discuss some implication of the Hopf algebra structure of the ring of quasi-symmetric functions corresponding to the Schur multiple zeta functions. This is a joint work with O. Phuksuwan and Y. Yamasaki.

廣嶋 透也 (Hiroshima Toya) 大阪大学

Crystal interpretation of a formula on the branching rule of C_n type

$V_q(\lambda)$ を支配的整ウェイト λ（Young 図形 λ と同一視）の有限次元既約 $U_q(\mathfrak{g})$ 加群（$\mathfrak{g} = \mathfrak{sp}(2n, \mathbb{C})$）とするとき、テンソル積 $V_q(\mu) \otimes V_q(\nu)$ の既約分解に現れる $V_q(\lambda)$ の多重度

$$[V_q(\mu) \otimes V_q(\nu) : V_q(\lambda)] = d^{\lambda}_{\mu\nu}$$

は、Littlewood-Richardson(LR) 係数を用いて、$l(\mu) + l(\nu) \leq n$ のとき（$l(\lambda)$ は Young 図形 λ の深さ)、

$$d^{\lambda}_{\mu\nu} = \sum_{\xi,\zeta,\eta \in \mathcal{P}} c^{\lambda}_{\xi\zeta} c^{\mu}_{\zeta\eta} c^{\nu}_{\eta\xi} \tag{1}$$

と書けることがわかっている（$\mathcal{P}$ は図形全体の集合).

本発表では、(1) 式を LR クリスタルで表現する次の全射を構成し、その証明の概略を紹介する．

$$\mathbf{B}_n(\nu)^{\lambda}_{\mu} \twoheadrightarrow \coprod_{\xi,\zeta,\eta \in \mathcal{P}} \mathbf{B}^{(+)}_n(\xi)^{\lambda}_{\zeta} \times \mathbf{B}^{(-)}_n(\eta)^{\mu}_{\zeta} \tag{2}$$

(2) 式で、左辺の $\mathbf{B}_n(\nu)^{\lambda}_{\mu}$ は C_n 型の LR クリスタルであり、右辺の $\mathbf{B}^{(+)}_n(\xi)^{\lambda}_{\zeta}$ と $\mathbf{B}^{(-)}_n(\eta)^{\mu}_{\zeta}$ は A_{n-1} 型の LR クリスタルである．

以上は C_n 型（$\mathfrak{g} = \mathfrak{sp}(2n, \mathbb{C})$）の場合であるが、$B_n$ 型（$\mathfrak{g} = \mathfrak{so}(2n+1, \mathbb{C})$）および D_n 型（$\mathfrak{g} = \mathfrak{so}(2n, \mathbb{C})$）の場合でも、安定領域、$l(\mu) + l(\nu) \leq n$ で (1) 式と同じ関係が成立する．本発表では更に、B_n 型および D_n 型の LR クリスタルが安定領域のもとで C_n 型

のものと同一であり、従って (2) 式が B_n 型および D_n 型においても成立し、(1) 式の表現を与えることを示す.

藤田 遼 (**Fujita Ryo**) 京都大学

Tilting modules in affine highest weight categories

Affine highest weight category, introduced by Kleshchev, is a generalization of the notion of highest weight category. For example, some module categories over central completions of (degenerate) affine Hecke algebras of GL (more generally, quiver Hecke algebras of finite Lie types) and polynomial current Lie algebras, and Soergel's deformed BGG category $\mathcal{O}$ are known to be affine highest weight. In this talk, we discuss tilting modules in affine highest weight categories. For a highest weight category, Ringel proved the existence of a special kind of tilting module which has both a standard filtration and a costandard filtration. We have a generalization of the Ringel's result for an affine highest weight category with a large categorical center. As a corollary, we obtain a simple criterion for an exact functor between two affine highest weight categories to give an equivalence. We can apply this criterion to prove that the Arakawa-Suzuki functor gives a fully faithful embedding of a block of BGG category $\mathcal{O}$ of $\mathfrak{gl}_m$ into the module category of the degenerate affine Hecke algebra of GL_n.

柳田 伸太郎 (**Yanagida Shintarou**) 名古屋大学

Torus skein algebra and mirror symmetry

We introduce the category $\mathcal{E}_q$ which is an F_1-analogue of the category of coherent sheaves over elliptic curve defined over a finite field. Although our category is not an abelian category, even nor an exact category, it is an example of so-called quasi-exact categories. Thus we can consider the Ringel-Hall algebra associated to $\mathcal{E}_q$. The category $\mathcal{E}_q$ also has an A_∞ structure, which can be seen as an F_1-analogue of Fukaya category of torus. The main statement is that the Ringel-Hall algebra of $\mathcal{E}_q$ is isomorphic to the skein algebra of torus. Our result recovers the recent work of Morton and Samuelson on the skein algebra and the elliptic Hall algebra.

八尋 耕平 (Yahiro Kohei) 東京大学

Integral transforms of D-modules on G-varieties

Beilinson and Bernstein established a relationship between D-modules on the full flag variety G/B and representations of the Lie algebra Lie G. They introduced intertwining functors, which are defined as integral transforms along G-orbits on the product $G/B \times G/B$ and gave a proof of Casselman's submodule theorem. In this talk, we discuss the partial flag case. We define intertwining functors as integral transforms along G-orbits of products of partial flag varieties satisfying some condition. We show that intertwining functors are equivalences of categories of D-modules on partial flag varieties. As an application of this result, we shows that similar construction for a G-variety X gives rise to equivalences between categories of D-modules on X equivariant with respect to parabolic subgroups.

渡邊 英也 (Watanabe Hideya) 東京工業大学

Crystal basis theory for a quantum symmetric pair

In the past two decades, quantum symmetric pairs have been recognized to be important in many fields of mathematics, such as Macdonald polynomials, Schur-Weyl duality, Lie superalgebras, Kazhdan-Lusztig theory, categorifications, and so on. Among them, we focus on the Schur-Weyl duality in type B. Let $\mathbf{U} = \mathbf{U}_n = U_q(\mathfrak{sl}_n)$ be the quantum group of type A, and $\mathbf{U}^x = \mathbf{U}^x_r \subset \mathbf{U}$ a coideal subalgebra which forms a quantum symmetric pair $(\mathbf{U}, \mathbf{U}^x)$ of type AIII, where $x = \imath$ if $n = 2r$ and $x = \jmath$ if $n = 2r+1$. The Schur-Weyl duality in type B means that $\mathbf{U}^x$ and the Hecke algebra $\mathcal{H}(W_d)$ of type B_d satisfy the double centralizer property on the d-th tensor power of the vector representation of $\mathbf{U}$; this result is due to Huanchen Bao and Weiqiang Wang. This shows that the representation theory of $\mathbf{U}^x$ is related to that of $\mathcal{H}(W_d)$. In this talk, we classify all the irreducible $\mathbf{U}^x$-modules in a suitable category, and then we associate each of them an analog of Kashiwara's crystal basis, which we call x-crystal basis. Since the x-crystal bases have a nice combinatorial property, we can solve some representation-theoretical questions by purely combinatorial ways.