

# RIMS Workshop

## “Aspects of Combinatorial Representaion Theory”

Date October 9 (Tue.) – 12 (Fri.)

Venue Room 111 (1st floor), Research Institute for Mathematical Sciences, Kyoto University

Organizer Daisuke Sagaki (University of Tsukuba)

Oct. 9 (Tue.)

13:30 – 14:30 Sota Asai (Nagoya University)

Bricks over preprojective algebras and join-irreducible elements in Coxeter groups

■ アブストラクト. Mizuno gave an isomorphism of lattices from a Coxeter group of Dynkin type to the set of torsion-free classes in the module category of the corresponding preprojective algebra. Combining it with my result on semibricks, we obtain a bijection from the Coxeter group to the set of semibricks over the preprojective algebra. My aim is to explicitly describe the semibrick associated to each element in the Coxeter group in this bijection. In this process, a combinatorial notion “canonical join representations” introduced by Reading, is very useful. I observed that the canonical join representation of an element in the Coxeter group gives the decomposition of the corresponding semibrick into bricks. I will talk about a combinatorial algorithm to determine the semibrick.

14:45 – 15:45 Diogo Kendy Matsumoto (Shibaura Institute of Technology)

Quiver-theoretical approach to dynamical Yang-Baxter maps

■ アブストラクト. 2005年に瀧川陽一(北海道大学)は、ヤン・バクスター写像(=ヤン・バクスター方程式の集合論的解)にパラメータを付加することによりダイナミカル・ヤン・バクスター写像を導入した。また、ダイナミカル・ヤン・バクスター写像の圏論的な枠組みとして、ダイナミカル集合の圏と呼ばれるテンソル圏が瀧川自身の手により与えられている。

本講演では、このダイナミカル集合の圏が箭のなす圏に埋め込めることを示し、ダイナミカル・ヤン・バクスター写像が箭論的ヤン・バクスター写像として表現できることを述べる。本発表の内容は、清水健一(芝浦工業大学)との共同研究に基づいている。

16:00 – 17:00 Teruhisa Tsuda (Hitotsubashi University)

Birational Weyl group actions via mutation combinatorics in cluster algebras

■ アブストラクト. 団代数は、箭(有向グラフ)の mutation と呼ばれる操作と、ある簡単な双有理変換によって生成される代数構造である。本講演では、大久保直人氏と増田哲氏(青山大)との共同研究に基づいて、団代数を介したワイル群の双有理表現の構成法について紹介する。得られる双有理表現は、ある有理代数多様体上のワイル群作用や  $q$ -差分パルヴェ方程式など、可積分系にも関係した興味深いクラスを与える。とくに、構成の鍵となる「閉路グラフに付随する鏡映変換」の組合せ的な側面に焦点をあてて論ずる。

Oct. 10 (Wed.)

9:30 – 10:30

Ayumu Hoshino (Hiroshima Institute of Technology)

Macdonald polynomials of type  $C_n$  with one-column diagrams and deformed Catalan numbers

■ アブストラクト. We present an explicit formula for the transition matrix  $\mathcal{C}$  from the type  $C_n$  degeneration of the Koornwinder polynomials  $P_{(1^r)}(x|a, -a, c, -c|q, t)$  with one column diagrams, to the type  $C_n$  monomial symmetric polynomials  $m_{(1^r)}(x)$ . The entries of the matrix  $\mathcal{C}$  enjoy a set of certain three term recursion relations, which can be regarded as a  $(a, c, t)$ -deformation of the one for the Catalan triangle or ballot numbers. Some transition matrices are studied associated with the type  $(C_n, C_n)$  Macdonald polynomials  $P_{(1^r)}^{(C_n, C_n)}(x|b; q, t) = P_{(1^r)}(x|b^{1/2}, -b^{1/2}, q^{1/2}b^{1/2}, -q^{1/2}b^{1/2}|q, t)$ . It is also shown that the  $q$ -ballot numbers appear as the Kostka polynomials, namely in the transition matrix from the Schur polynomials  $P_{(1^r)}^{(C_n, C_n)}(x|q; q, q)$  to the Hall-Littlewood polynomials  $P_{(1^r)}^{(C_n, C_n)}(x|t; 0, t)$ . This is a joint work with Jun'ichi Shiraishi (University of Tokyo).

10:45 – 11:45

Sho Matsumoto (Kagoshima University)

Kerov polynomials for spin representations of symmetric groups

■ アブストラクト. 対称群の既約指標を正規化したものは、自由キウムラントを変数とする多項式で表すことができる。この多項式は Kerov 多項式と呼ばれ、対称群の漸近的表現論において重要な役割を果たす。ここで自由キウムラントとは自由確率論の用語だが、Young 図形から定まる推移測度のモーメントを通じて定義される量である。Kerov 多項式の係数はすべて非負整数であることが Kerov 自身により予想され、Féray(2009) により証明された。本講演では対称群のスピンの既約指標に対し、対応する「スピン Kerov 多項式」をどのように定めるべきかを考察し、Kerov 予想のスピンの版を提示する。arXiv:1803.01121.

13:30 – 14:30

Yoshitaka Toyozawa (Okayama University) · Takeshi Suzuki (Okayama University)

On enumeration of cylindric standard tableaux

■ アブストラクト. skew Young 図を平行移動しながら周期的に張り合わせて得られる図を巡回的 skew Young 図 (cylindric diagram), その上の”標準盤”を巡回的標準盤 (cylindric standard tableau) と呼ぶ。巡回的標準盤は表現論的には (退化) アフィン Hecke 代数の特別なクラスの既約モジュラー表現の基底を記述することが知られており、また、可解模型の理論等に現れるレベル制限付標準盤とも自然に同一視される。本公演では、巡回的標準盤の個数およびその母関数に関するいくつかの公式を紹介し、巡回的組合せ論の立場からの証明についても言及する。また、近年成瀬氏によって得られた skew Young 図上の標準盤の個数に関するフック公式の拡張を予想として提示する。

14:45 – 15:45

Soichi Okada (Nagoya University)

Multivariate skew hook formula for  $d$ -complete posets

■ アブストラクト. In 1954, Frame, Robinson and Thrall gave a nice product formula for the number of standard tableaux on a given Young diagram in terms of hook lengths. Recently Naruse finds a subtraction-free formula for the number of standard tableaux on a skew Young diagram in terms of excited diagrams and hook lengths. On the other hand, Proctor introduced a wide class of posets, called  $d$ -complete posets, as a generalization of Young diagrams and shifted Young diagrams, and obtained in collaboration with Peterson a product formula for the multivariate generating function of poset partitions on a  $d$ -complete poset  $P$ , which implies the hook formula for the number of standard tableaux on  $P$ . In this talk, we introduce the notion of excited diagrams for  $d$ -complete posets, and present a subtraction-free formula (Naruse-type skew hook formula) for the multivariate generating functions for poset partitions on a  $d$ -complete poset with an order filter removed.

This talk is based on a joint work with Hiroshi Naruse (University of Yamanashi).

16:00 – 17:00

Satoshi Naito (Tokyo Institute of Technology)

Pieri-Chevalley formula in the equivariant K-theory of semi-infinite flag manifolds

■ アブストラクト. We give a Pieri-Chevalley formula for anti-dominant weights in the torus-equivariant K-group of a semi-infinite flag manifold, which describes the (tensor) product of the class of a line bundle with the class of the structure sheaf of a semi-infinite Schubert variety, in terms of the semi-infinite analog of Lakshmibai-Seshadri paths. As an application, we obtain a Monk formula in the K-group above, which describes the multiplication by the class of the structure sheaf of a semi-infinite Schubert variety of codimension one. On the basis of the isomorphism between the K-group above and the torus-equivariant (small) quantum K-group of an ordinary flag manifold, which has recently been established by Syu Kato, our results yield an explicit description of the quantum product by a line bundle associated to an anti-dominant fundamental weight; in particular, in type A, we can verify a conjectural Monk formula (presented by Lenart and Postnikov) in the quantum K-theory of a flag manifold. This talk is based on a joint work with D. Sagaki and D. Orr.

Oct. 11 (Thu.)

9:30 – 10:30

Ryosuke Kodera (Kobe University)

Braid group action on affine Yangian

■ アブストラクト. ヤンギアンへのブレイド群作用を導入し、その性質を調べる。応用として、アフィン A 型ヤンギアンのハイゼンベルグ部分代数の構成について述べる。

- 10:45 – 11:45 Masao Ishikawa (Okayama University)  
Several classes of plane partitions with the same generating function
- アブストラクト. Mills-Robbins-Rumsey の数えた種々の平面分割や descending plane partition 等の母関数について重みを変えて考察したい. 交代符号行列の母関数と比較して様々な予想ができる.
- 13:30 – 14:30 Motoki Takigiku (University of Tokyo)  
Automorphisms on the ring of symmetric functions and stable and dual stable Grothendieck polynomials
- アブストラクト. We show that the dual stable Grothendieck polynomials  $g_\lambda$  and their sums  $\sum_{\mu \subset \lambda} g_\mu$  (which represent  $K$ -homology classes of boundary ideal sheaves and structure sheaves of Schubert varieties in the Grassmannians) have the same product structure constants. More precisely, the linear endomorphism  $g_\lambda \mapsto \sum_{\mu \subset \lambda} g_\mu$  on the ring of symmetric functions is described as the operator  $F^\perp$ , the adjoint of the multiplication by  $F$ , by  $F = \sum_i h_i = \sum_\mu G_\mu$  where  $h_i$  is the complete symmetric function and  $G_\mu$  is the stable Grothendieck polynomial. Then the fact  $F^\perp$  is a ring morphism stems from that the coproduct of  $F$  is equal to  $F \otimes F$ . Next we give a generalization: starting with another such elements  $\sum_i h_i t^i$ , we obtain a deformation with a parameter  $t$  of the ring automorphism above, as well as identities involving stable and dual stable Grothendieck polynomials.
- 14:45 – 15:45 Hideya Watanabe (Tokyo Institute of Technology)  
Global crystal bases for integrable modules over a quantum symmetric pair of type AIII
- アブストラクト. Quantum symmetric pairs (QSPs) appear in many areas of mathematics and physics such as representation theory, low-dimensional topology, and integrable system. Especially, QSPs of type AIII are known to have a deep connection with the representation theory of Hecke algebras of type B. In this talk, we introduce the notion of global crystal bases for integrable modules over a QSP of type AIII (with asymptotic parameters), and their properties.

16:00 – 17:00 Naoki Fujita (Tokyo Institute of Technology)  
Divided difference operators on polytopes and polyhedral realizations of crystal bases

■ アブストラクト. The main object in this talk is a certain rational convex polytope whose lattice points give a polyhedral realization of a highest weight crystal basis. This is also identical to a Newton-Okounkov body of a flag variety, and it gives a toric degeneration. In this talk, we see that a specific class of this polytope is given by Kiritchenko's divided difference operators on polytopes. This implies that polytopes in this class are all lattice polytopes. As an application, we give a sufficient condition for the corresponding toric variety to be Gorenstein Fano.

Oct. 12 (Fri.)

9:30 – 10:30 Toshiyuki Abe (Ehime University)  
Extensions of tensor products of the VOA  $V_{\sqrt{2}A_n}^\sigma$

■ アブストラクト. The VOA  $V_{\sqrt{2}A_n}$  is the lattice vertex operator algebra associated with an even lattice  $\sqrt{2}A_n$ , where  $A_n$  is the rank  $n$  root lattice of type  $A$ . An  $(n+1)$ -cycle  $\sigma$  of the Weyl group  $W(A_n) \cong S_{n+1}$  lifts to an automorphism of  $V_{\sqrt{2}A_n}$  of order  $n+1$ . We describe extensions of tensor products of the orbifold model  $V_{\sqrt{2}A_n}^\sigma$  like as framed vertex operator algebras. This talk is based on a joint work with Ching Hung Lam and Hiromichi Yamada.

10:45 – 11:45 Kentaro Wada (Shinshu University)  
Finite dimensional simple modules of  $(q, \mathbf{Q})$ -current algebras

■ アブストラクト. A  $(q, \mathbf{Q})$ -current algebra  $U_q(\mathfrak{gl}_m^{\mathbf{Q}}[x])$  was introduced in a study of cyclotomic  $q$ -Schur algebras.  $U_q(\mathfrak{gl}_m^{\mathbf{Q}}[x])$  has parameters  $q$  and  $\mathbf{Q} = (Q_1, Q_2, \dots, Q_{m-1})$ . If we put  $q = 1$ , we obtain the universal enveloping algebra of the deformed current Lie algebra  $\mathfrak{gl}_m^{\mathbf{Q}}[x]$ , where  $\mathfrak{gl}_m^{\mathbf{Q}}[x]$  is a certain deformation of the polynomial current Lie algebra  $\mathfrak{gl}_m[x]$  associated with the general linear Lie algebra  $\mathfrak{gl}_m$  with deformation parameters  $\mathbf{Q} = (Q_1, \dots, Q_{m-1})$ .

In this talk, we discuss on finite dimensional simple modules of  $(q, \mathbf{Q})$ -current algebras.