Annulus twist of a knot and log transform

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Motoh Tange

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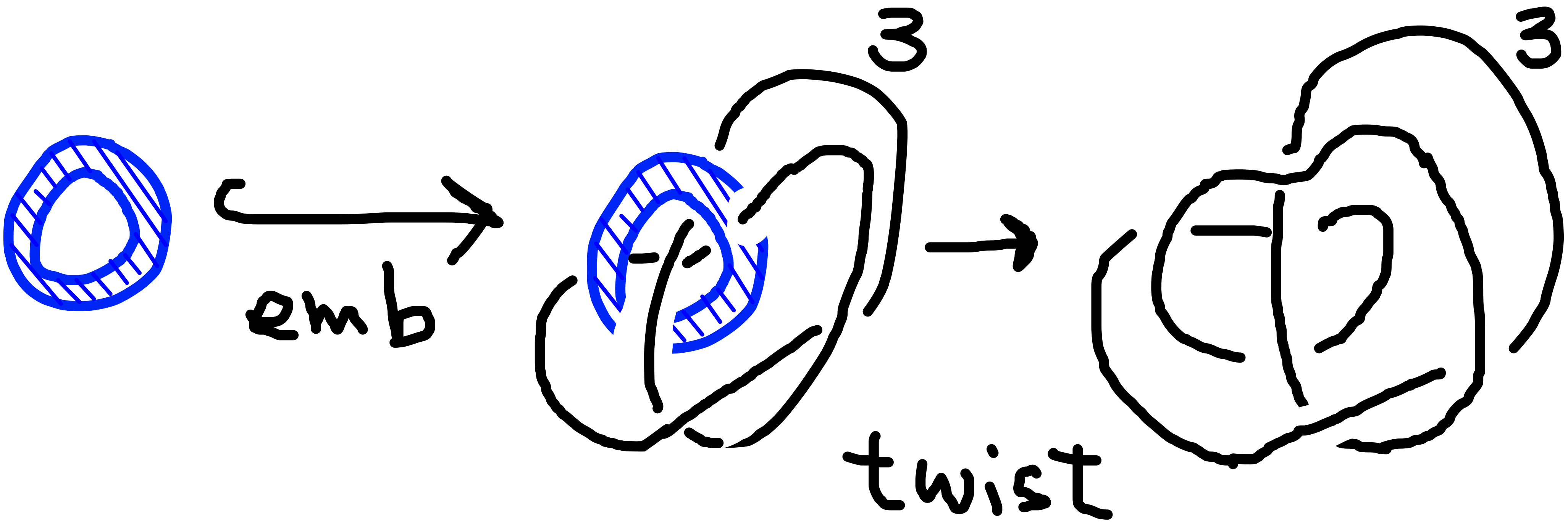
Annulus twist (by Osoinach)



Osoinach constructed infinitely many surgery presentations for a manifold.

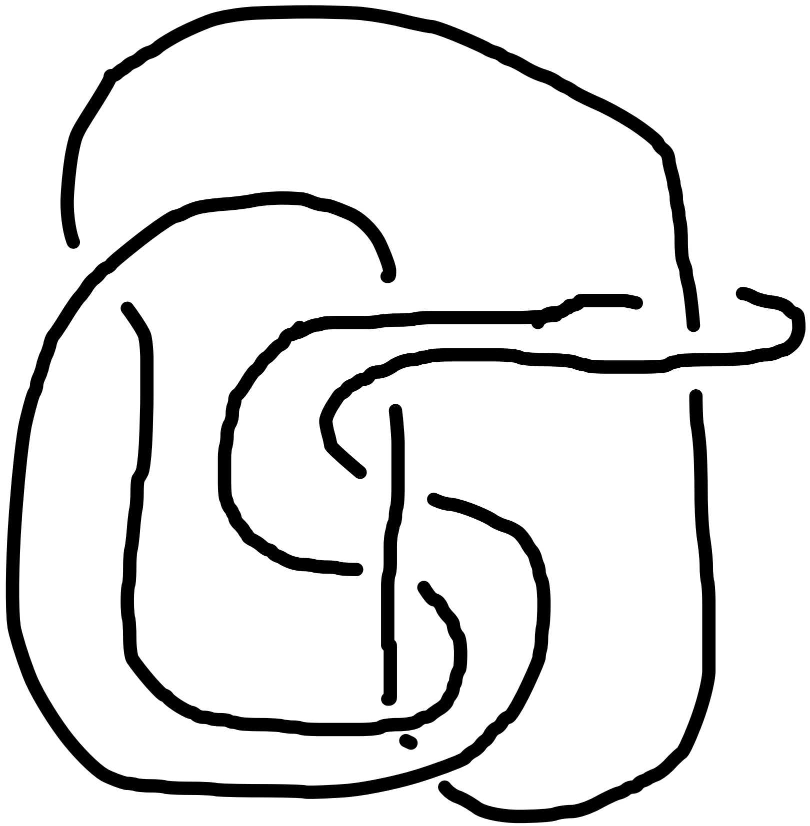
$$M(K_1, r) \cong M(K_2, r) \cong$$

surgery presentation.

Motivation

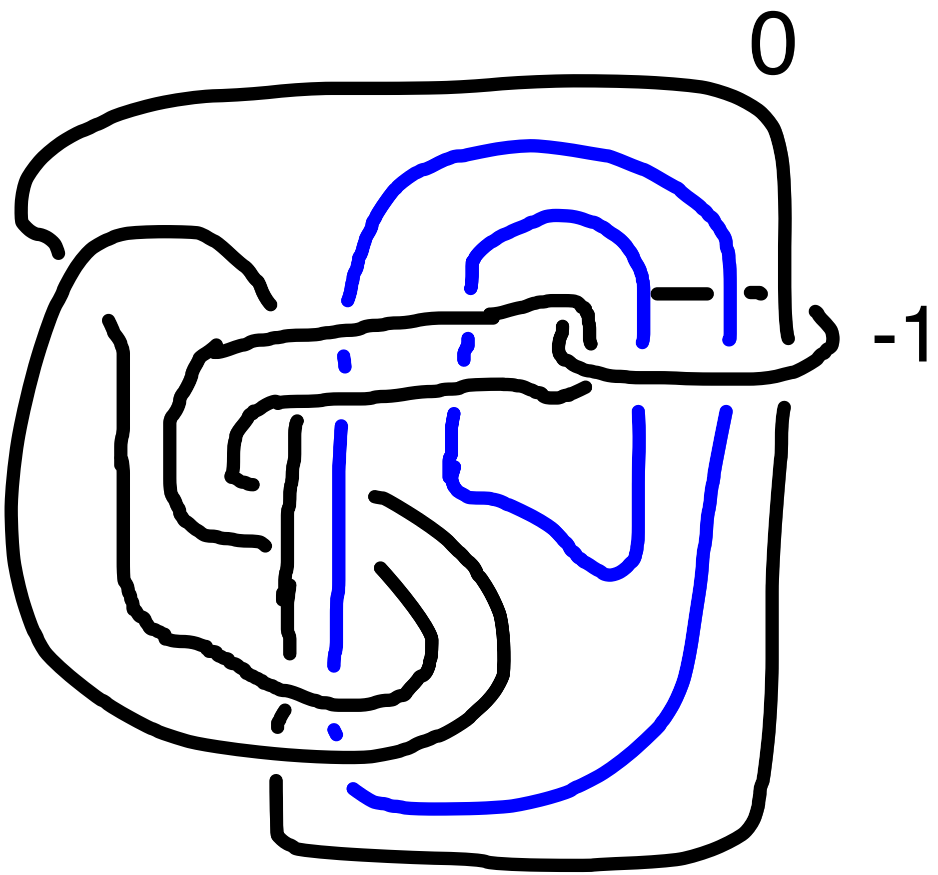
Abe's research

8_20



Unknotting number 1 and ribbon.

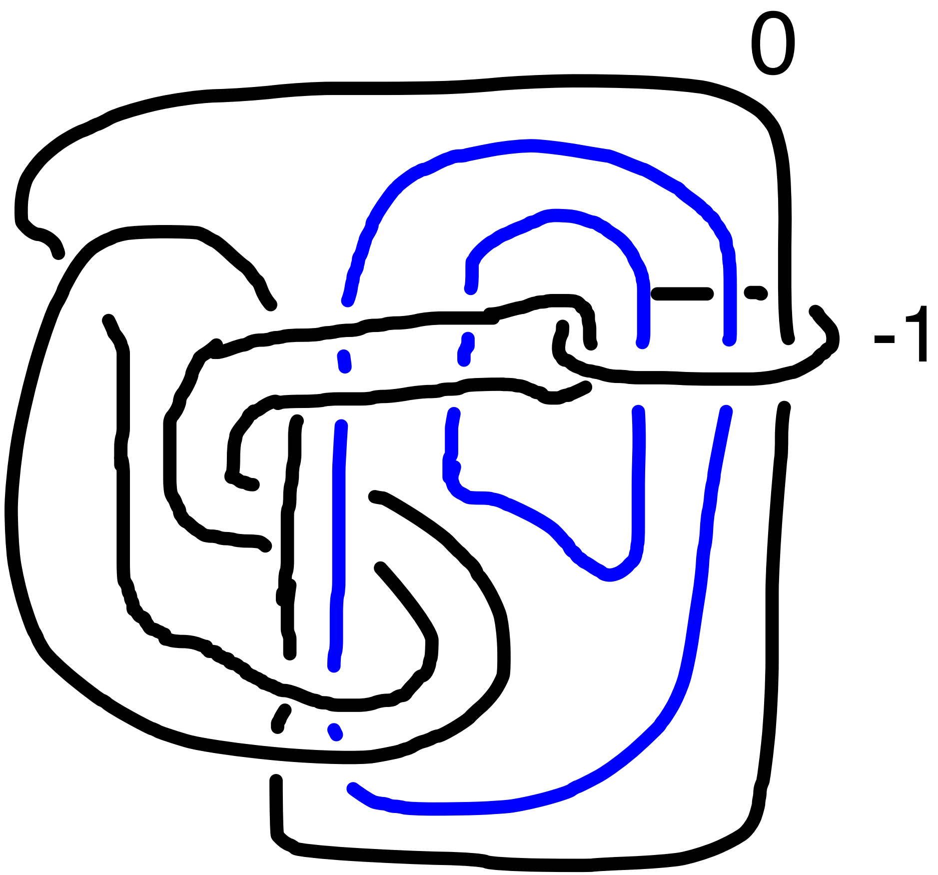
In particular, slice.



K_i

i times annulus twist of 8_{20}

$$M(8_{20},0) \cong M(K_1,0) \cong M(K_2,0)$$



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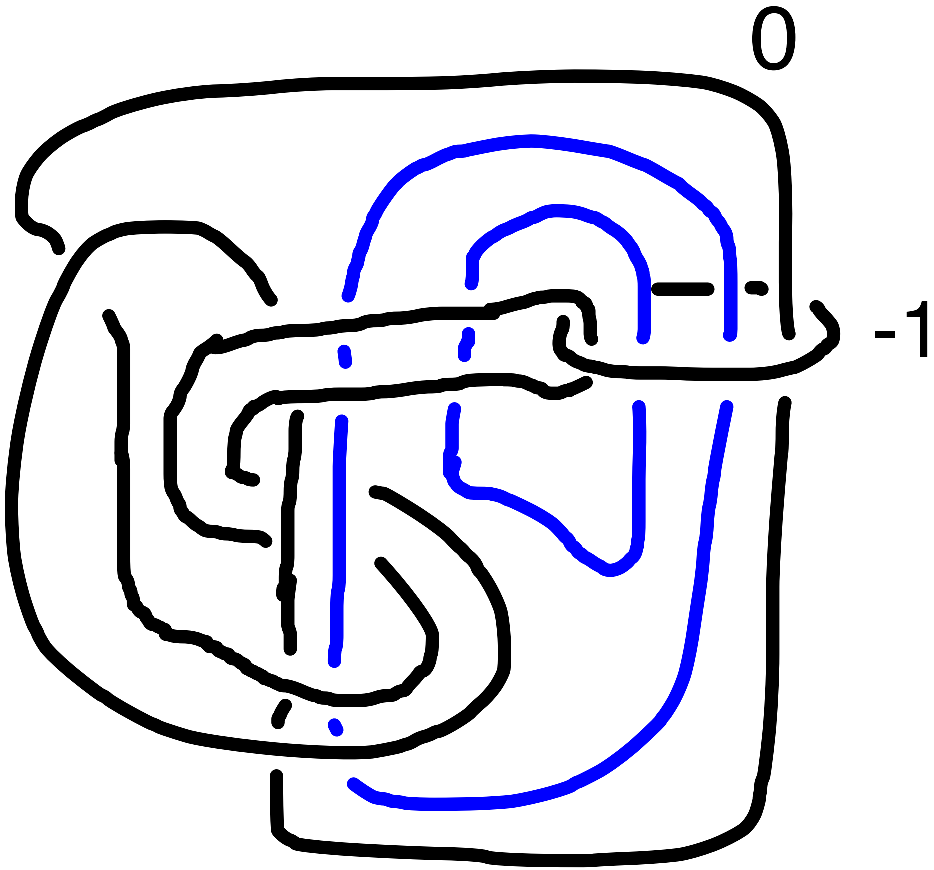

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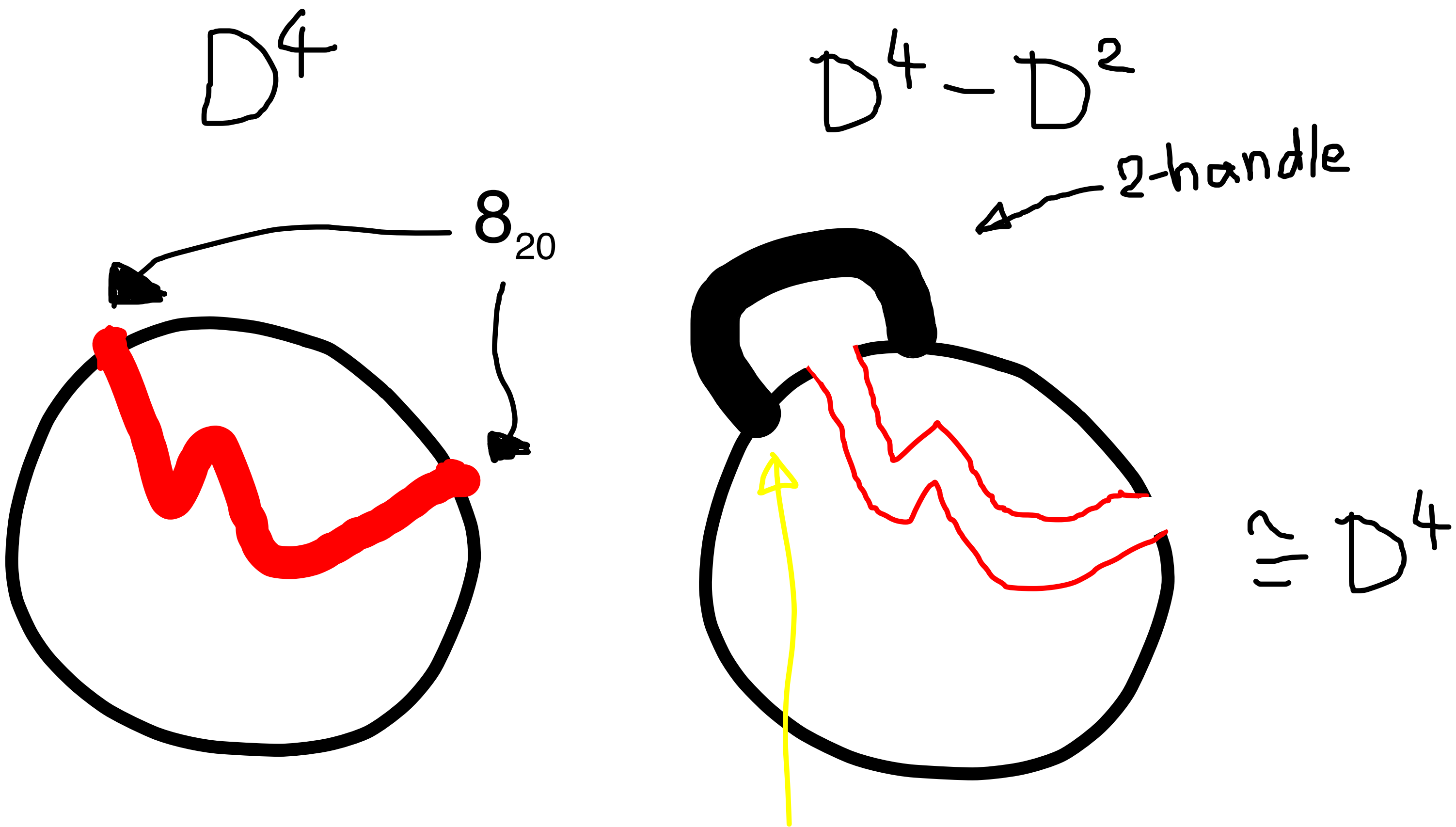
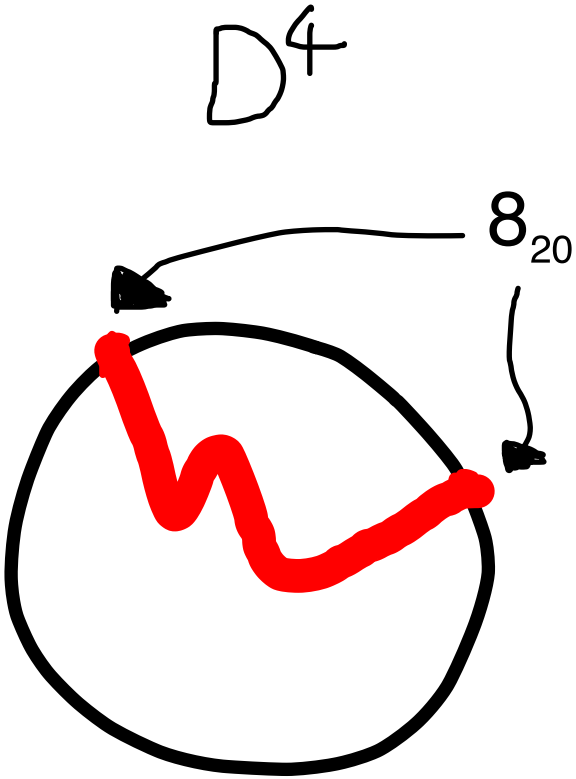
Furthermore,

$$\begin{array}{ccccc} M(8_{20},0) & \cong & M(K_1,0) & \cong & M(K_2,0) \\ \cap & & \cap & & \cap \\ X(8_{20},0) & \cong & X(K_1,0) & \cong & X(K_2,0) \end{array}$$



a ribbon disk of 8_{20} meridian of 8_{20}



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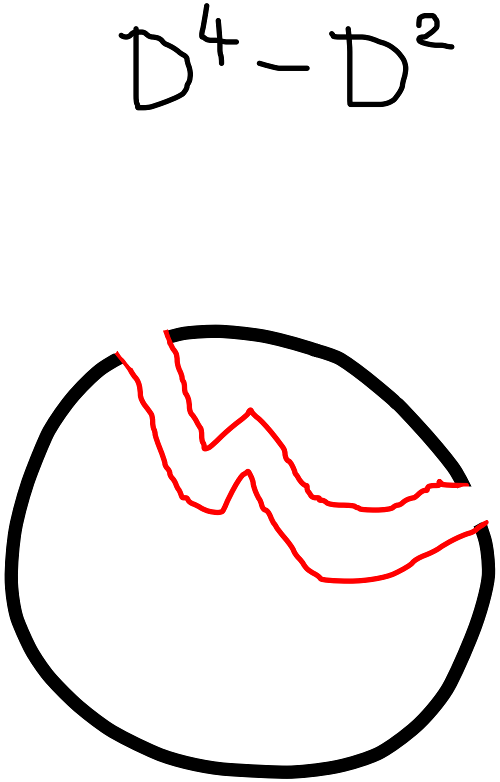
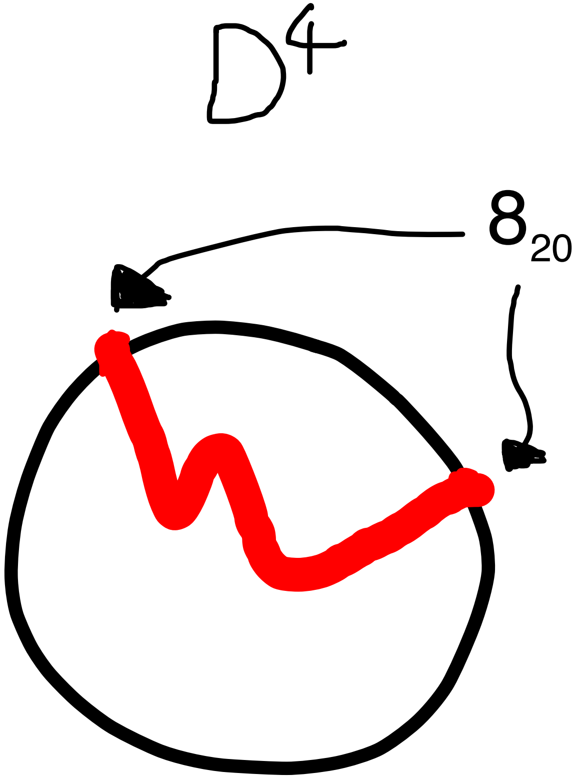


ribbon disk complement of 8_{20}

$$M(8_{20}, 0) \cong M(K_1, 0)$$



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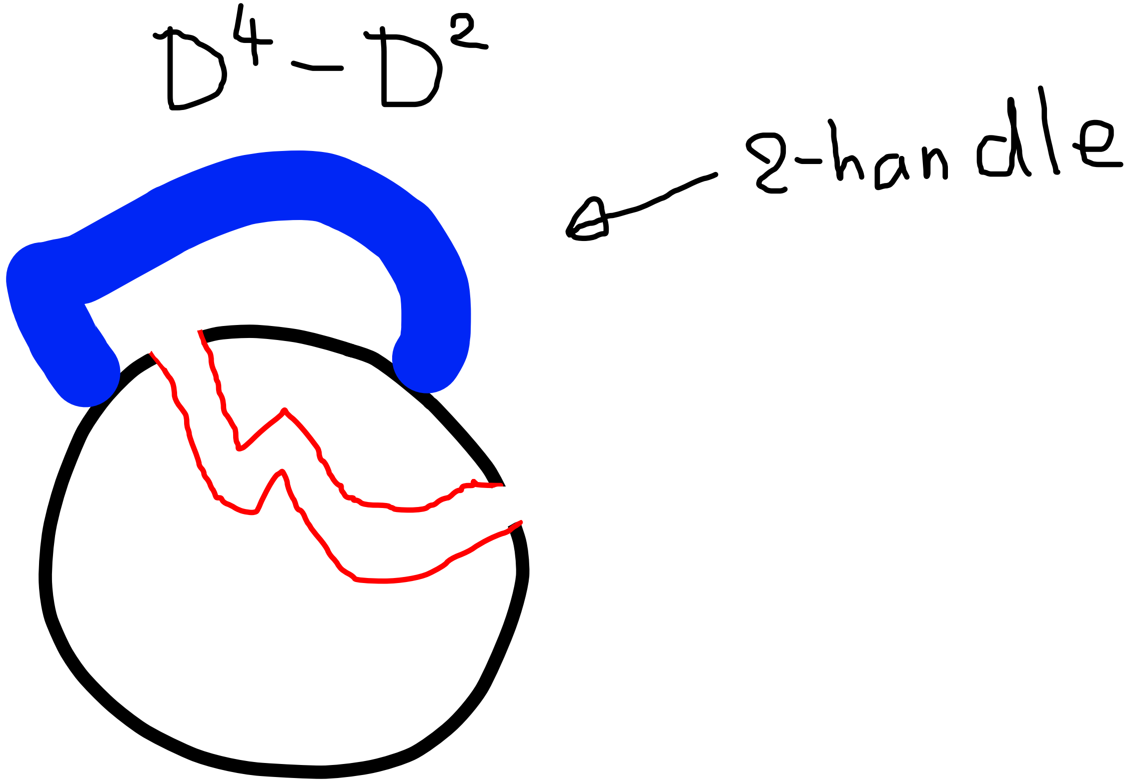

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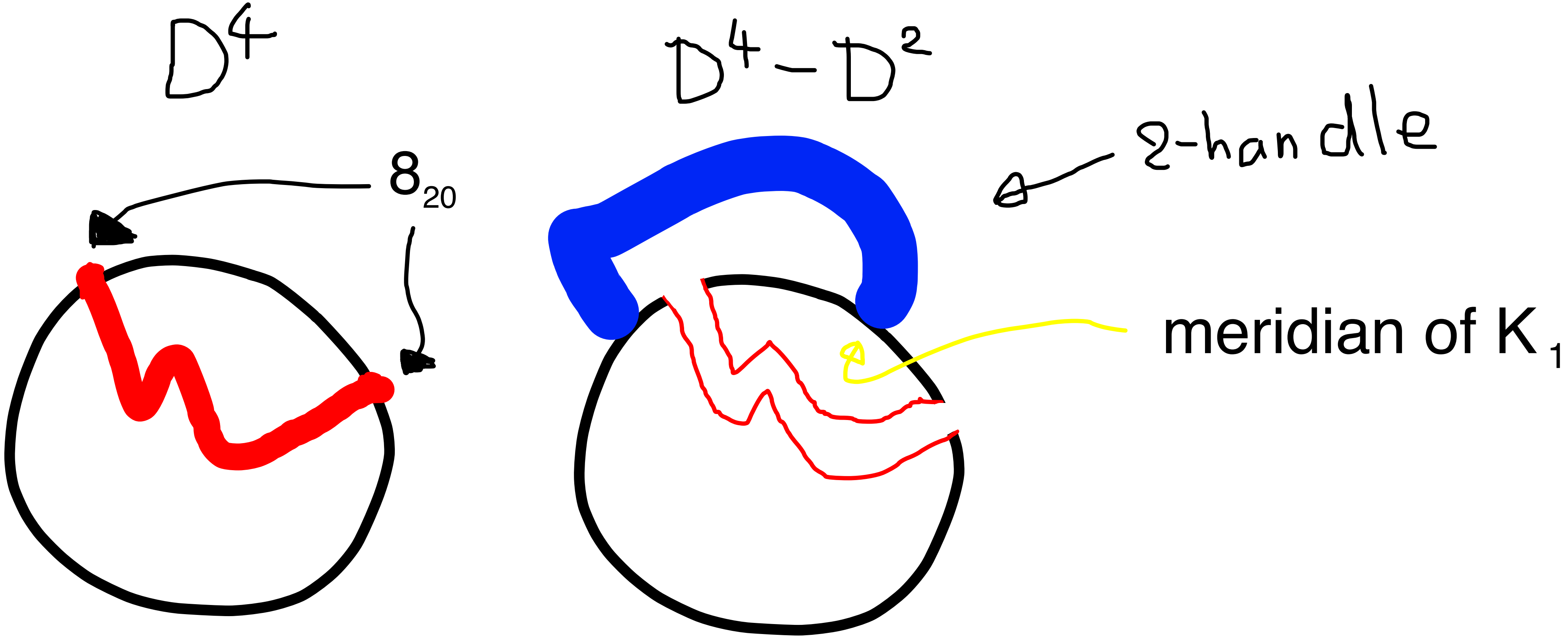

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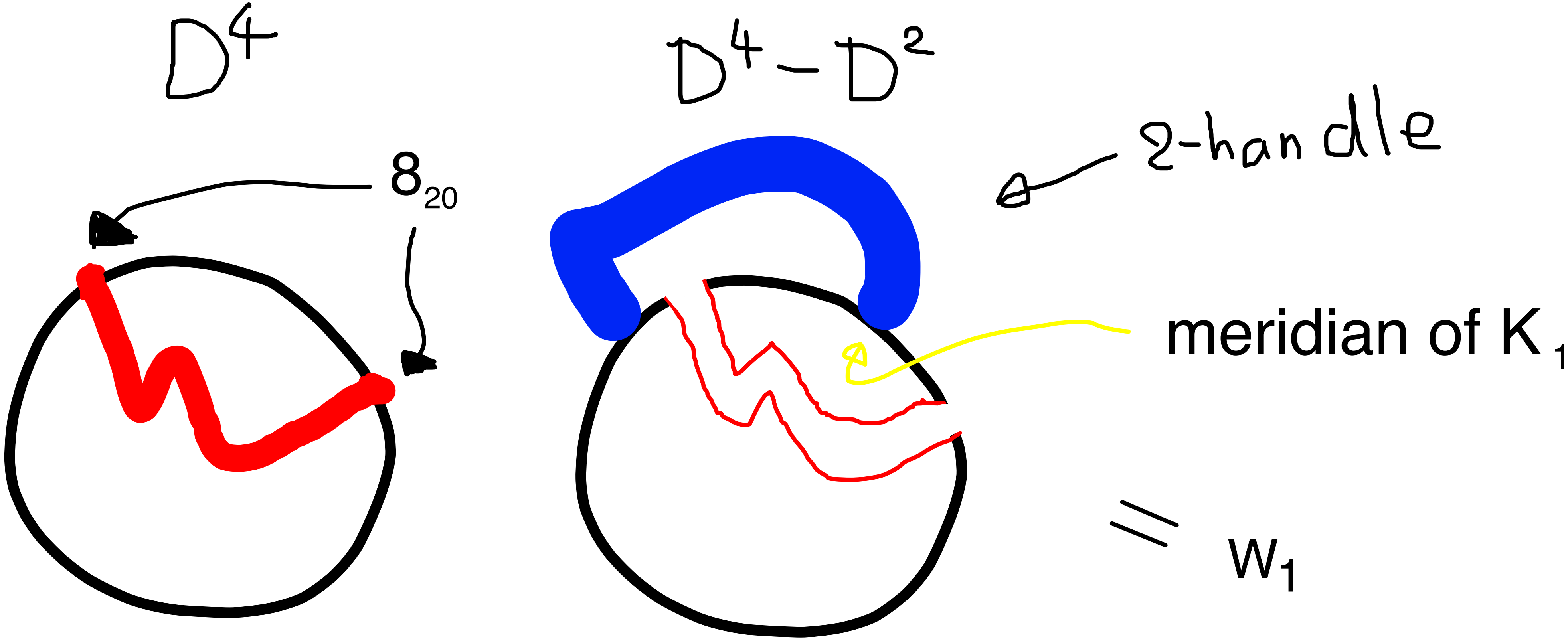

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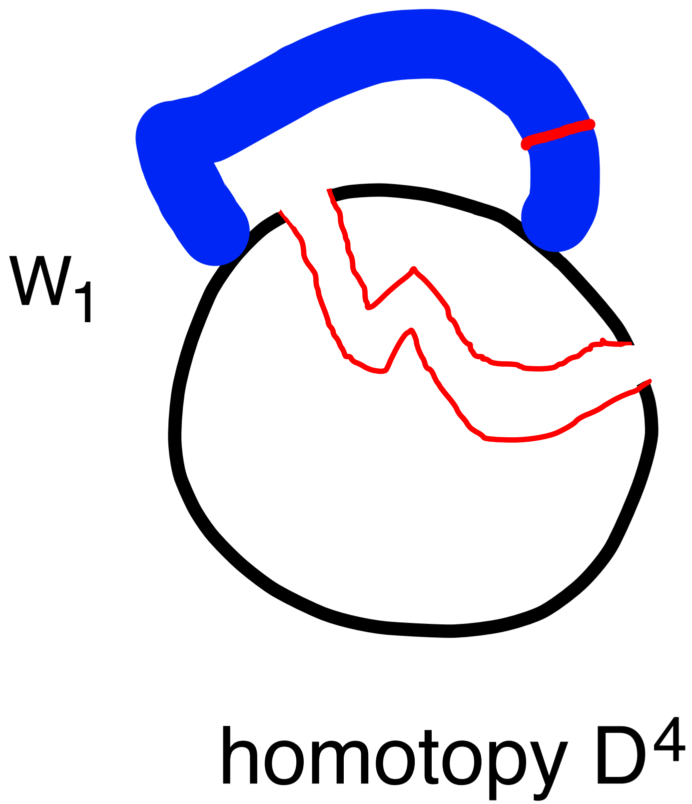
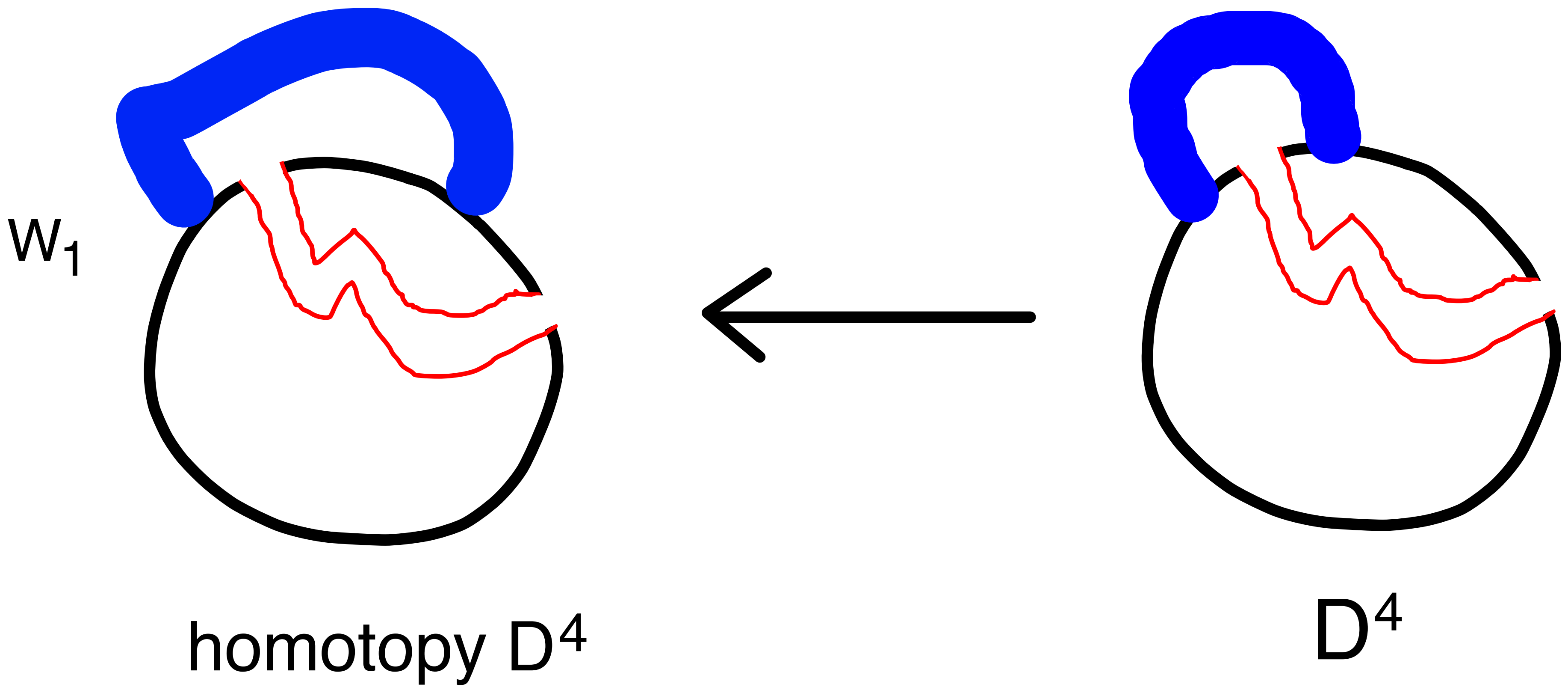
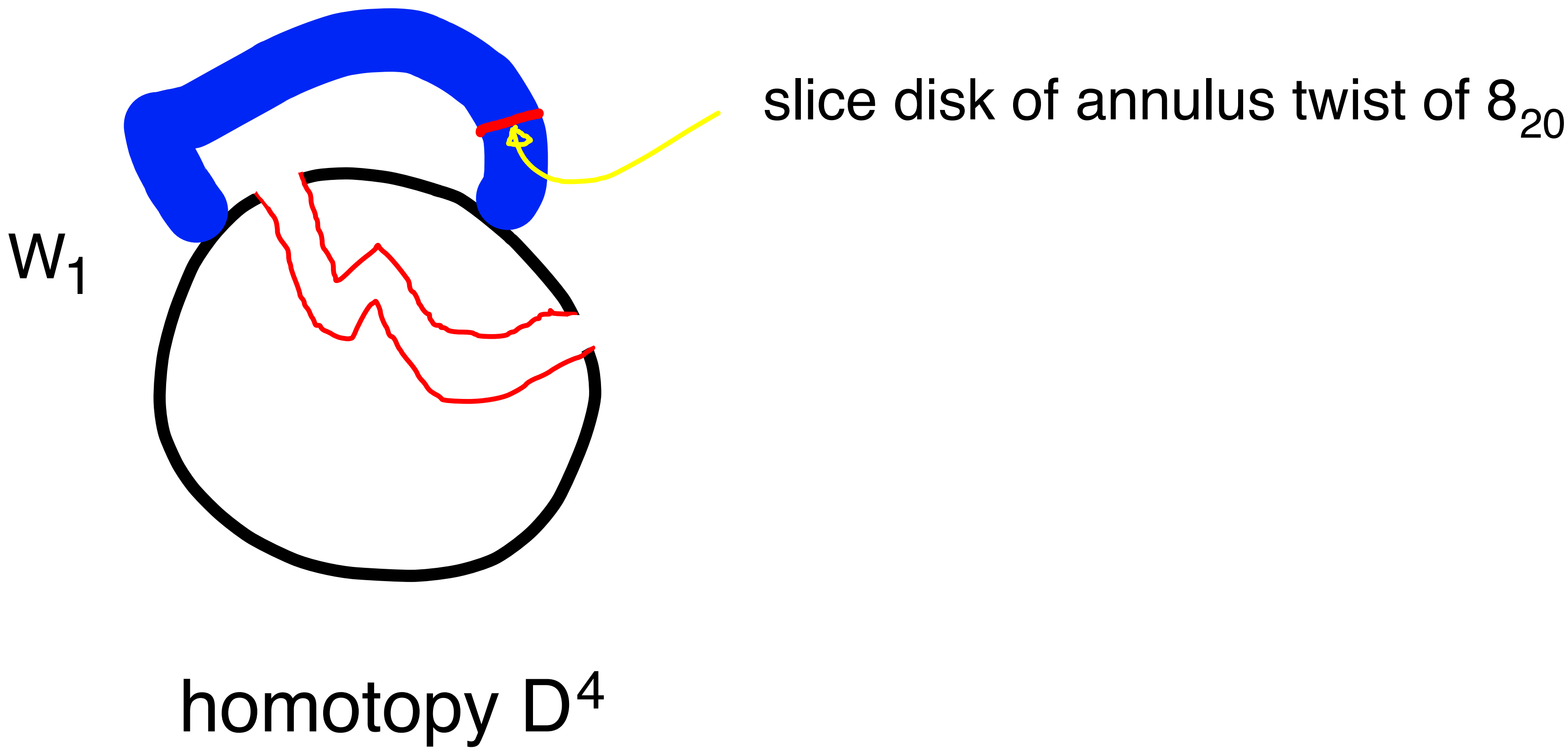

W_1
homotopy D^4


W1
homotopy D4
D4


slice disk of annulus twist of 8_{20}
W_1
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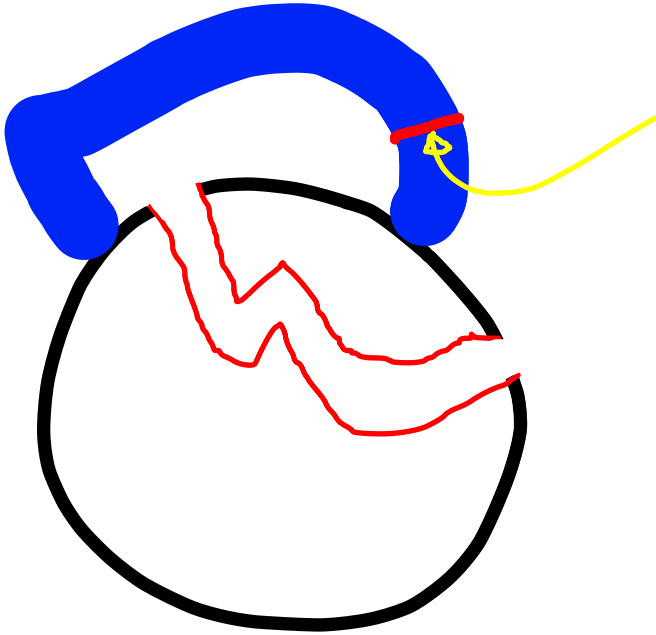
W_1

slice disk of annulus twist of 8_{20}

K_1 : the boundary is a knot in S^3

homotopy D^4

W_1



slice disk of annulus twist of 8_{20}

K_1 : the boundary is a knot in S^3

homotopy D^4

The main problem of Abe's talk in the previous this workshop is

Problem

Is K_1 a slice knot in S^3 ?

Theorem

W_n is diffeomorphic to the standard D^4.

Corollary

K_n is a slice knot in S^3.

Theorem

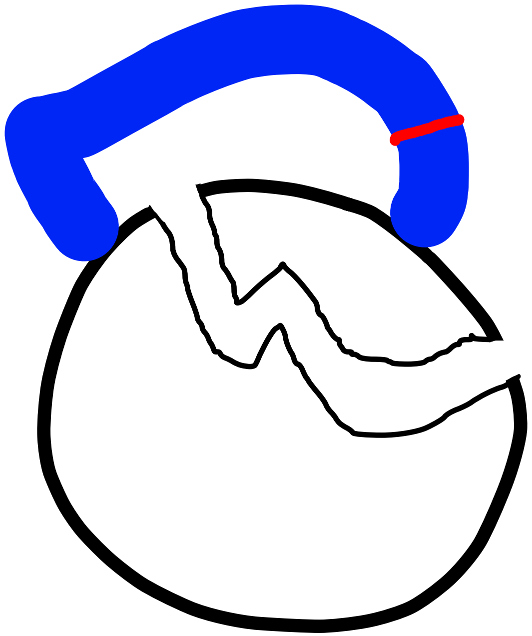
W_n is diffeomorphic to the standard D^4.

Corollary

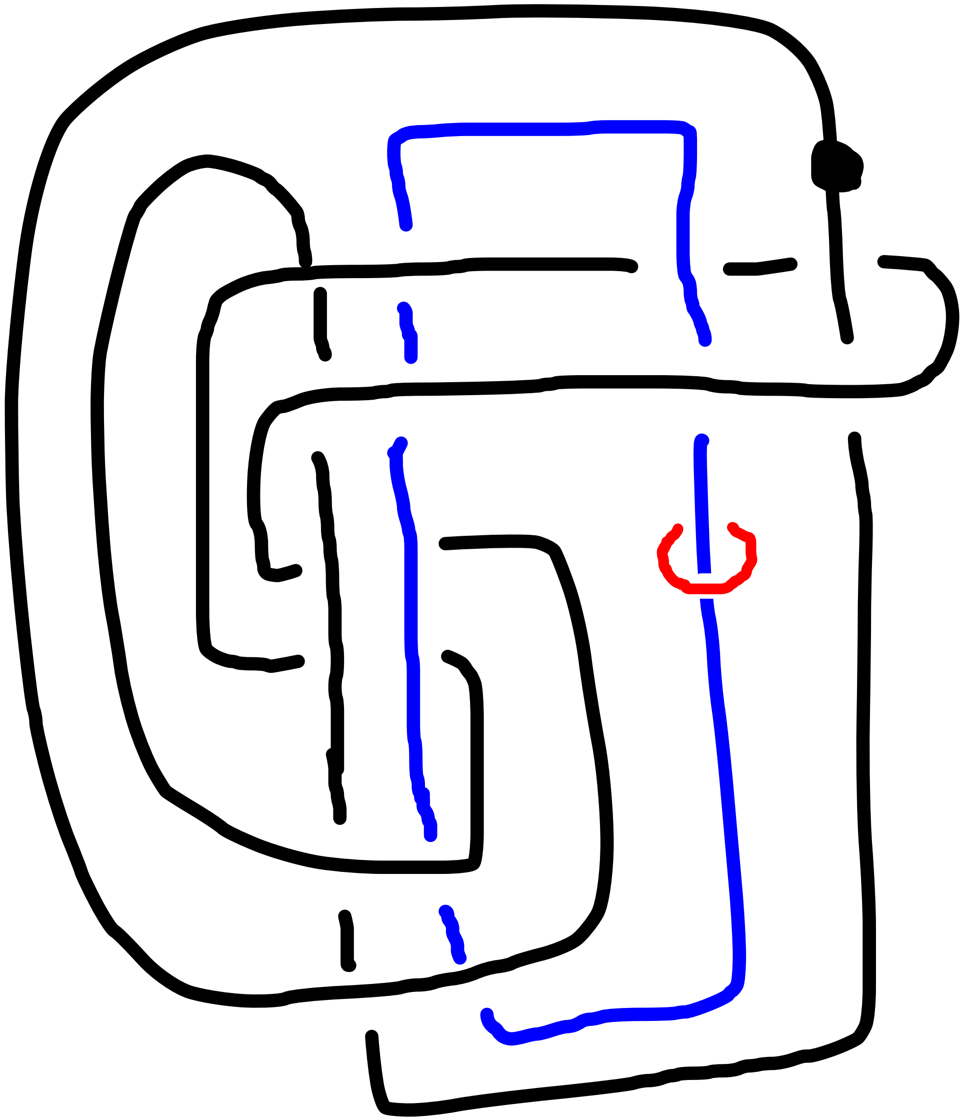
K_n is a slice knot in S^3.

This is the answer of the main problem of Abe's talk
"On the shake genus ....." in
Hiroshima Topology Conference Four Dimensional Topology

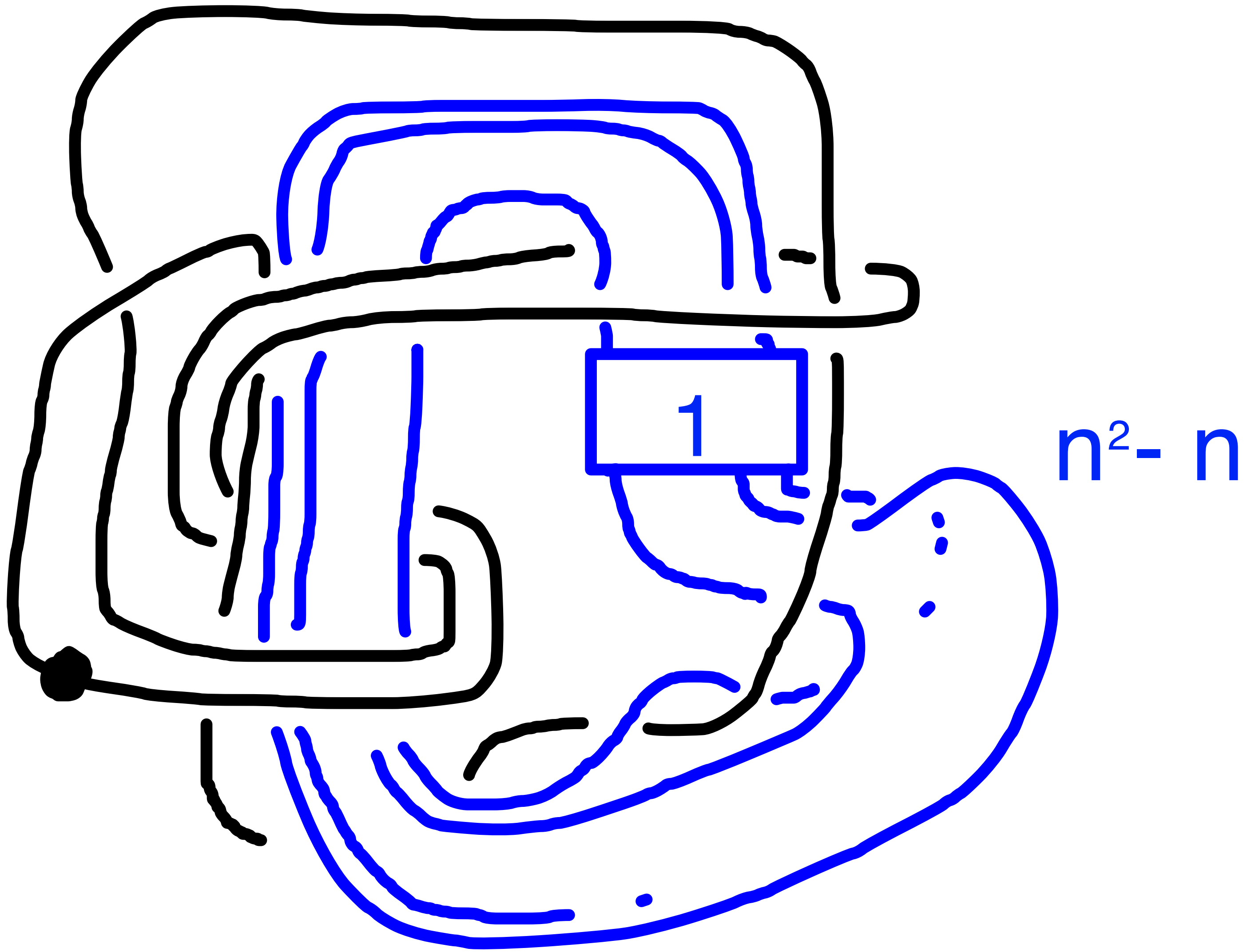
W_1



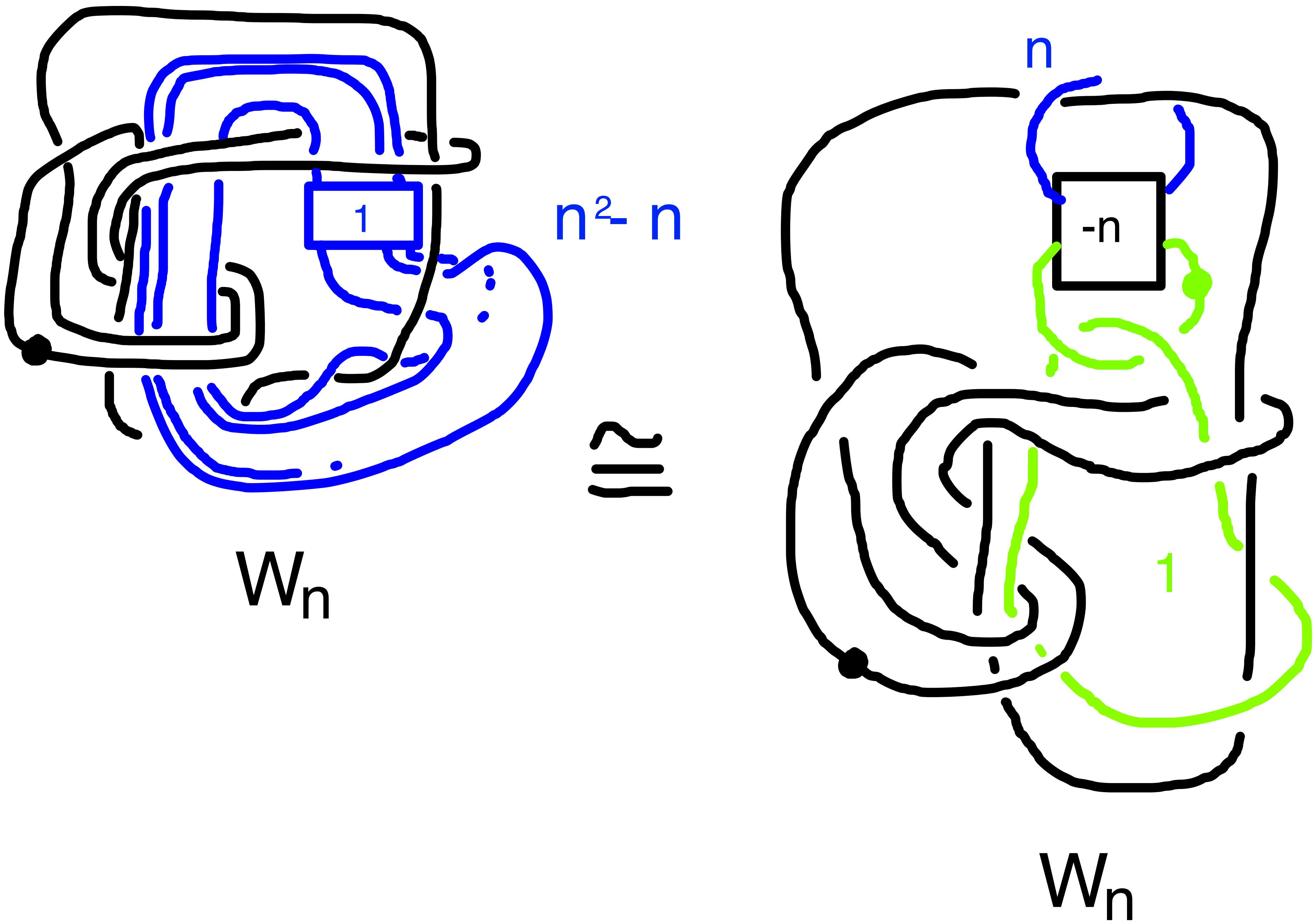
homotopy D^4



W_1



W_n



1
n² - n
W_n
≅
n
-n
1
W_n

Log transform

$T^2 \hookrightarrow X$ triv normal bdle

Log transform

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$$\boxed{[X - T^2 \times D^2] \cup T^2 \times D^2}$$

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log transform

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$$\boxed{[X - T^2 \times D^2] \cup T^2 \times D^2}$$

log transform

$\varphi \cdot T^2 \times \partial D^2 \to T^2 \times \partial D^2$

$$\boxed{\left[X - T^2 \times D^2\right] \cup T^2 \times D^2}$$

$$\varphi : T^2 \times \partial D^2 \to T^2 \times \partial D^2$$

$$\varphi(\partial D^2) = q\,\gamma + p\,\partial D^2$$

$\gamma \subset T^2$ curve

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$$\varphi(\partial D^2) = q\,\gamma + p\,\partial D^2$$

$\underline{\underline{\gamma}} \subset T^2$ curve

direction

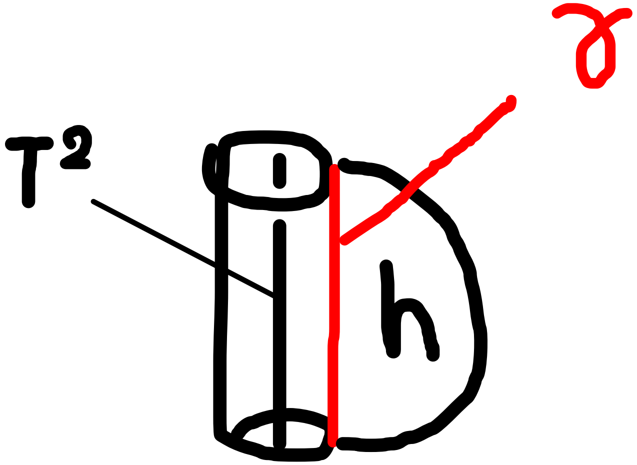
$$\boxed{[X - T^2 \times D^2] \cup T^2 \times D^2}$$

$\underline{\underline{\gamma}} \subset T^2$ curve

direction

$$X(T, p, \gamma, q)$$

Gompf's lemma



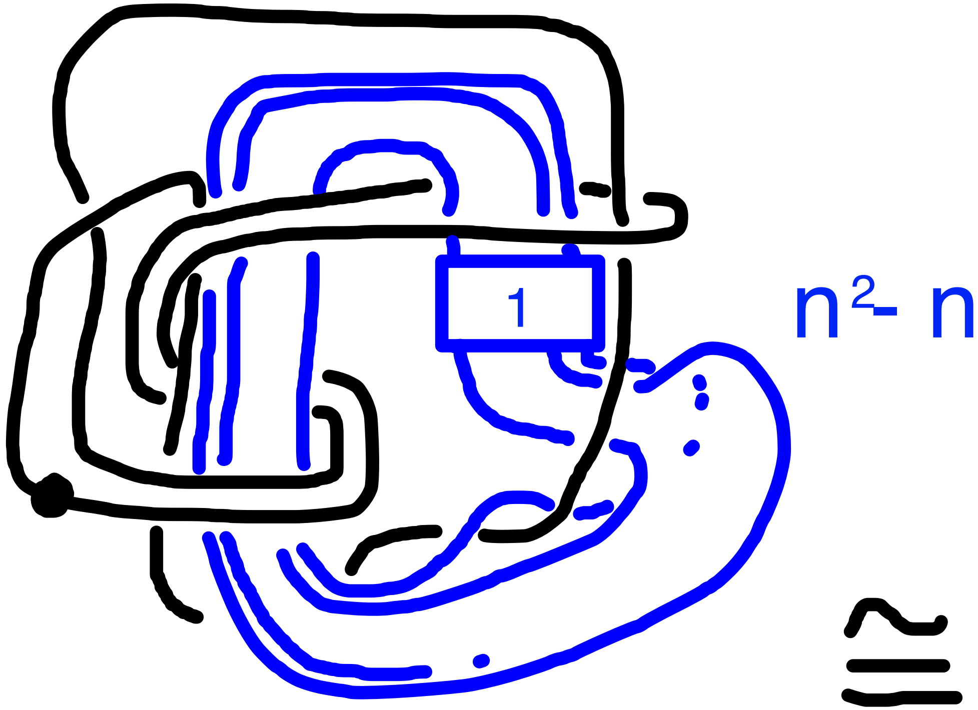
$$F \hookrightarrow X^4$$

$$F = T^2 \times D^2 \cup h^2$$

If log trans. of $T^2 \hookrightarrow F \hookrightarrow X$

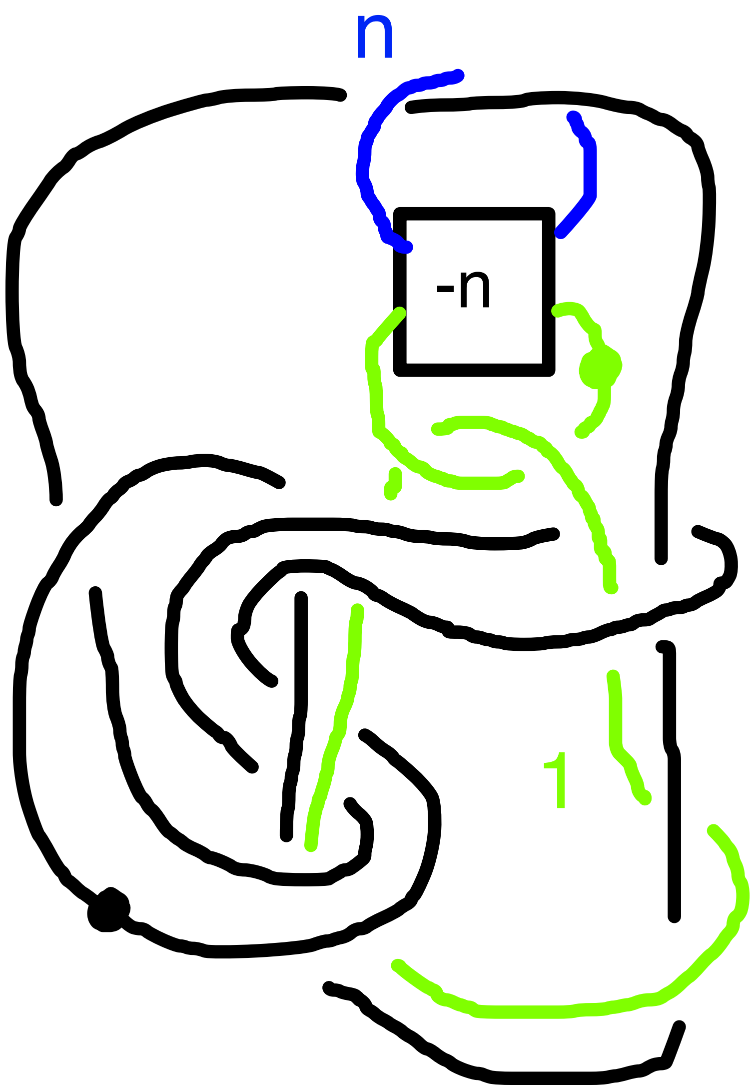
$$\Rightarrow X(T, 1, \gamma, n) = X \quad //$$

diffeo.

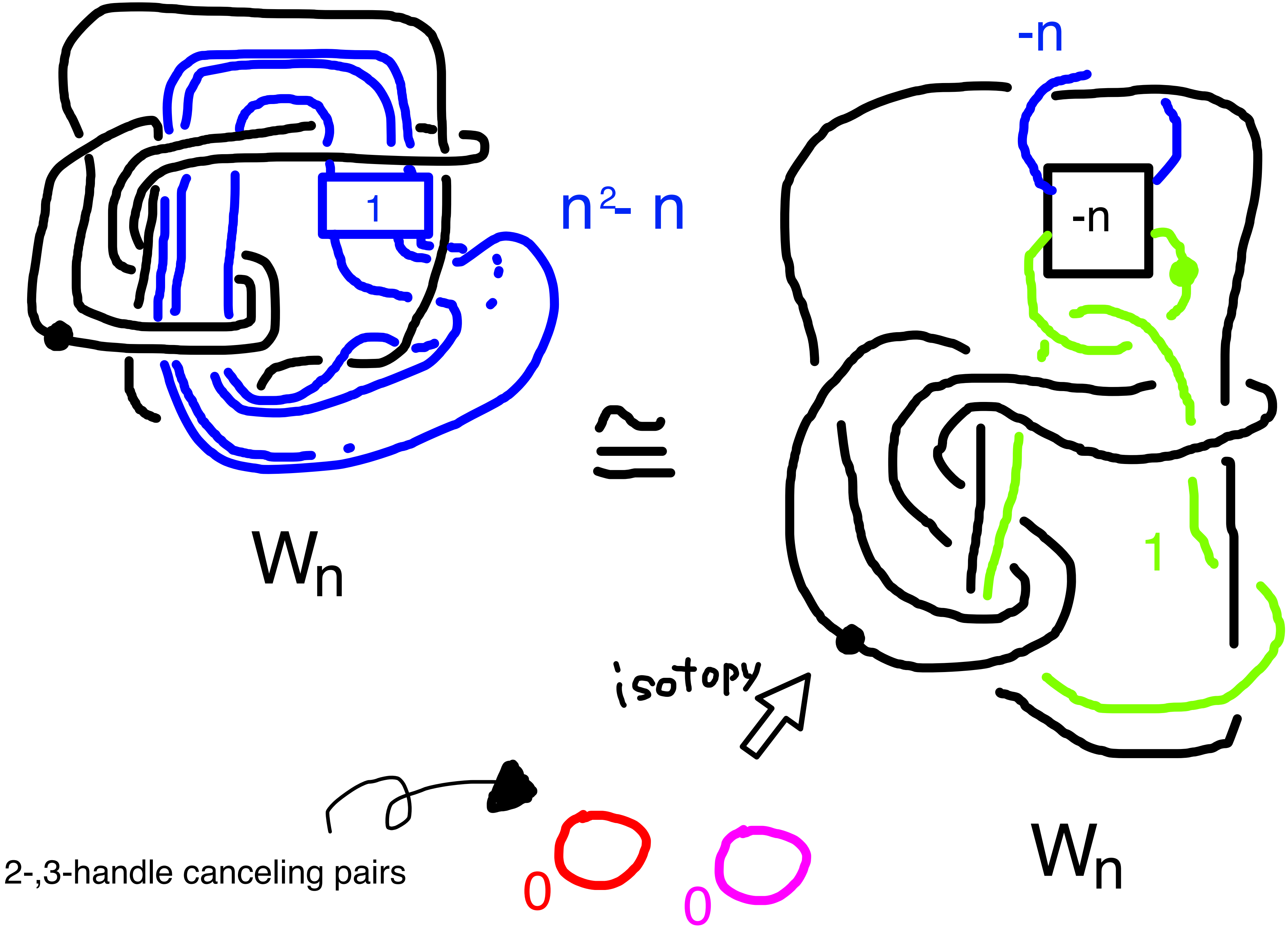


W_n

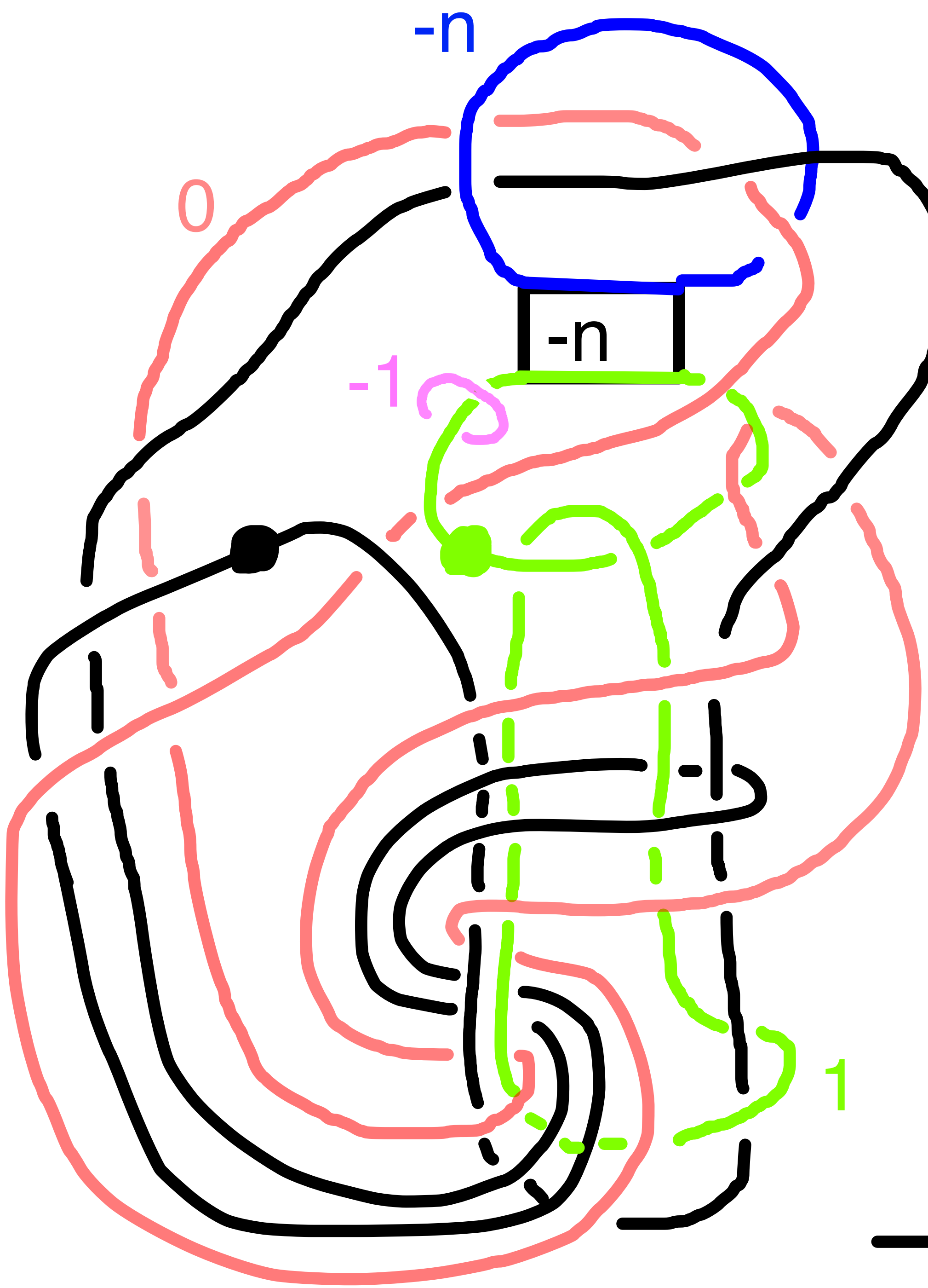
$\cong$



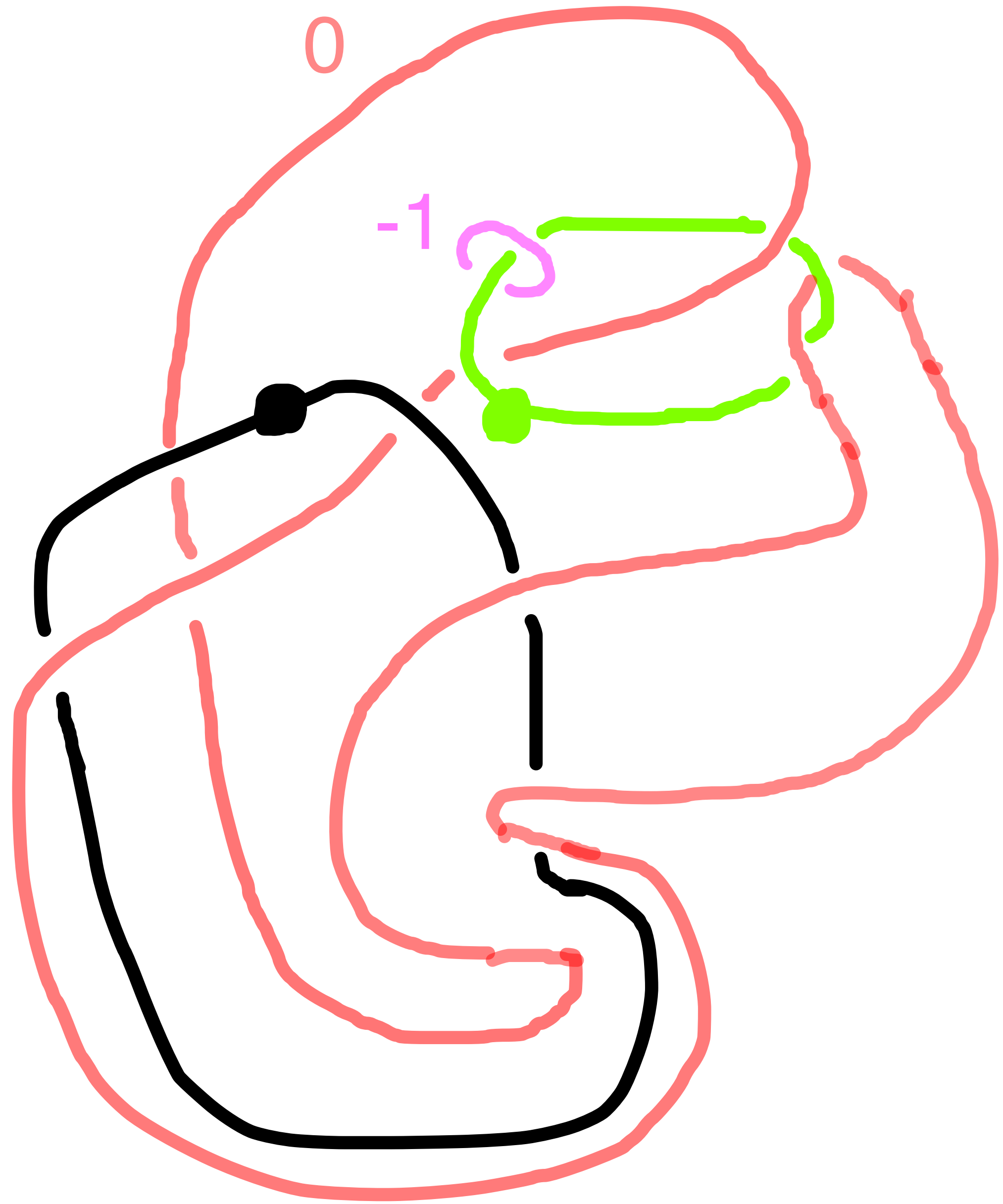
W_n

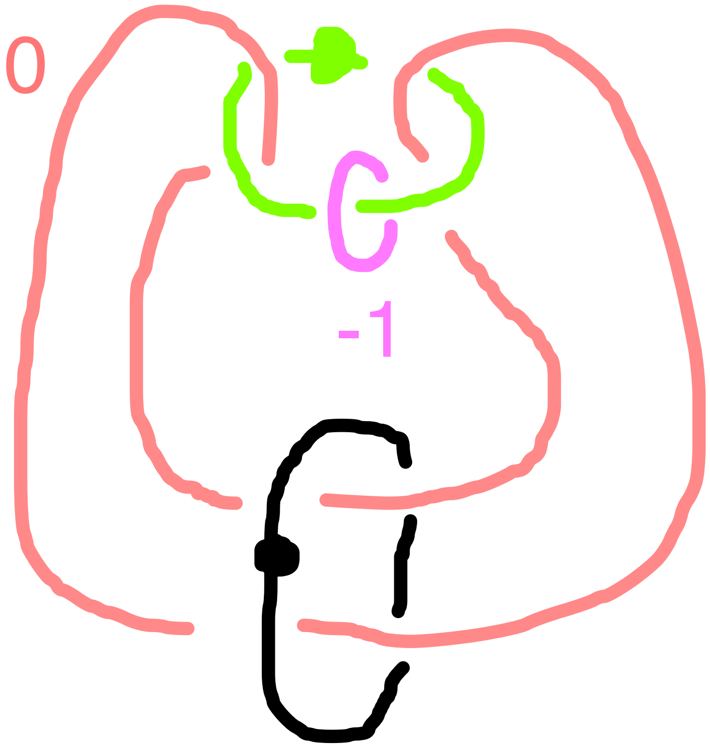


1
n²- n
≅
Wn
-n
-n
1
isotopy
Wn
2-,3-handle canceling pairs
0
0



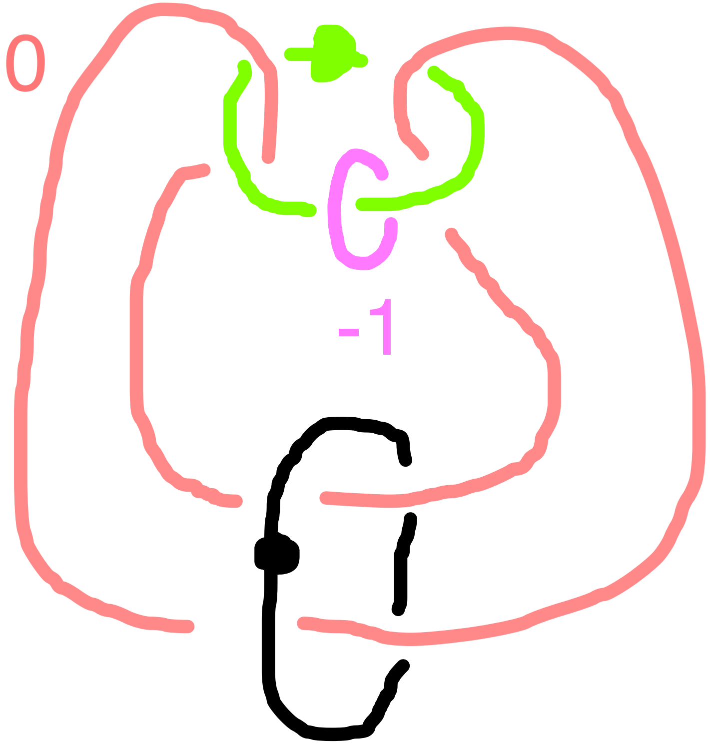
$\supset$





$= F$

$$T^2 \hookrightarrow F \hookrightarrow W_n$$



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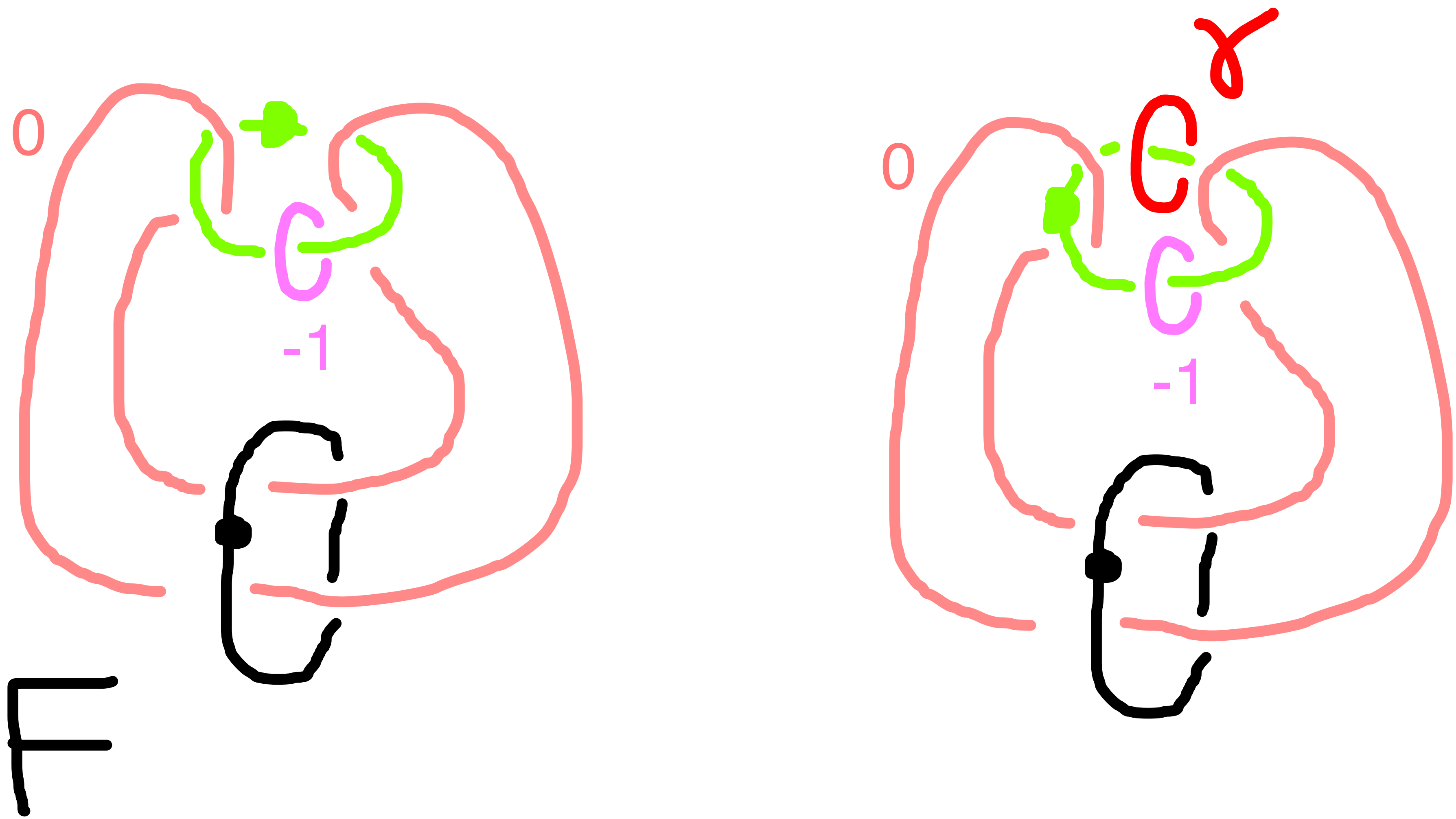
γ direction



$$T^2 \hookrightarrow F \hookrightarrow W_n$$

$$W_n(T, 1, \gamma, 1) = W_{n-1}$$

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Gompf's lemma

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$$\|$$

$$W_n$$

Gompf's lemma

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$$\|$$ Gompf's lemma

$$W_n$$

$$W_n = W_{n-1} = W_{n-2} = \cdots = W_0 = D^4$$

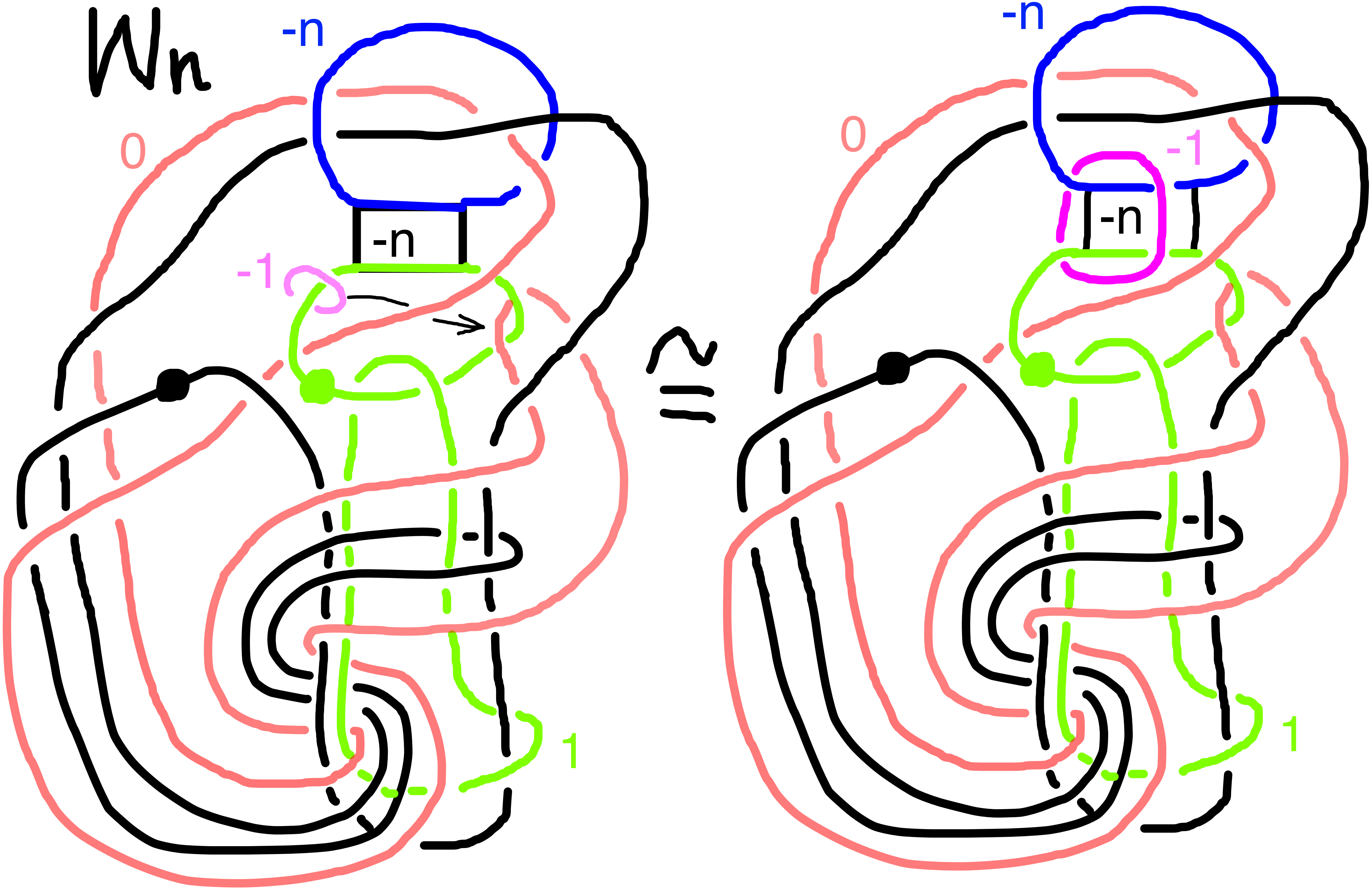
$$W_n(T, 1, \gamma, 1) = W_{n-1}$$

$$\begin{array}{c} W_n(T, 1, \gamma, 1) \\ \| \;\; \text{Gompf's lemma} \\ W_n \end{array}$$

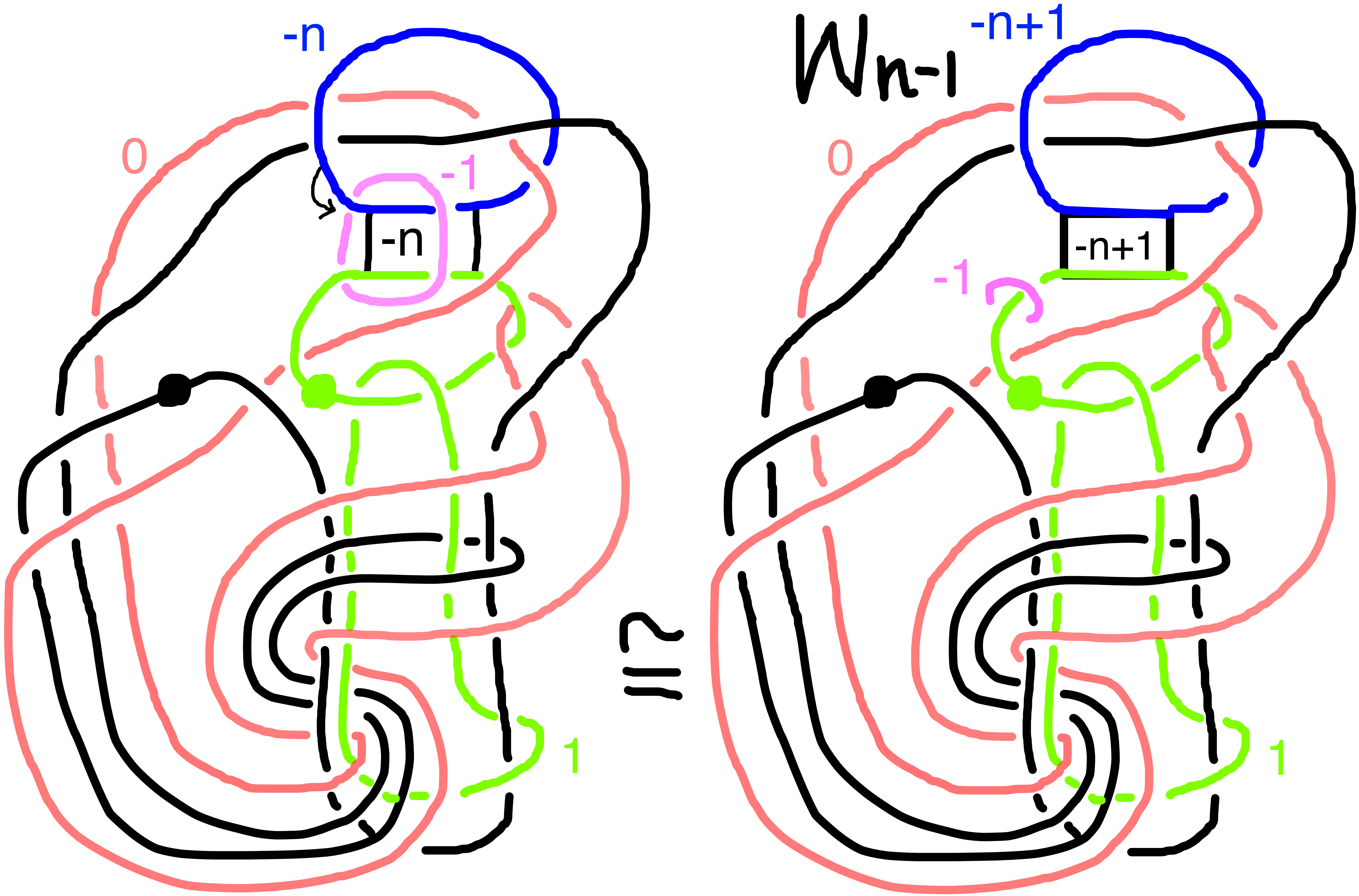
$$W_n = W_{n-1} = W_{n-2} = \cdots = W_0 = D^4$$

$\therefore$ W_n is the std ball

$\therefore$ K_n's are all slice.



W_n
-n
0
-n
-1
-1
1
≅
-n
0
-1
-n
-1
1



-n
0
-1
-n
1
≅
W_{n-1}
-n+1
0
-1
-n+1
1

Further,

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Akbulut proved.

Capell-Shaneson spheres

are standard S^4

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Akbulut proved. Σm

Cappell-Shaneson spheres

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He used 2,3 cancelling pair

Further,

Akbulut proved. Σ_m Cappell-Shaneson spheres are standard S^4

He use 2,3 cancelling pair

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & m+1 \end{pmatrix}$$

However,

$$T^2 \hookrightarrow F \overset{\exists}{\hookrightarrow} \Sigma_m$$

$$T^2 \hookrightarrow F \overset{\exists}{\hookrightarrow} \Sigma_n$$

$$\Sigma_n(T, 1, \gamma, 1) = \Sigma_{n-1}$$

$$\| \quad \text{Gompf's lemma}$$

$$\Sigma_n$$