数学外書輪講I

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注意-1.

1. １日３人程度
2. セミナーとは、書いてあることをわかりやすく解説をすること.
3. かならずノートを作り、発表のための準備をすること。もし、授業時間前までに準備をしていない場合は、次の人に回します。
4. 出席を取る.

問題-1-1. [英文和訳]
次の英文を和訳せよ.

(1) Euclidean n-space $\mathbb{R}^n$ is defined as the set of all n-tuples $(x^1, \cdots, x^n)$ of real numbers x^i (a "1-tuple of numbers" is just a number and $\mathbb{R}^1 = \mathbb{R}$, the set of all real numbers). An element of $\mathbb{R}^n$ is often called a point in $\mathbb{R}^n$, and $\mathbb{R}^1, \mathbb{R}^2, \mathbb{R}^3$ are often called the line, the plane, and space, respcetively. If x denotes an element in $\mathbb{R}^n$, then x is an n-tuple of numbers, the ith one of which is denoted x^i; thus we can write

$$x = (x^1, ..., x^n). \tag{1}$$

(2) Theorem: There are no four points in the plane such that the distance between each pair is an odd integer. (2)

(3) Theorem: A rectangle R with side lengths 1 and x, where x is irrational, cannot be tiled by finitely many squares (so that the squares have disjoint interiors and cover all of R). (2)

(4) Suppose that complete bipartite graphs $H_1, H_2, \cdots, H_m$ disjointly cover all edges of K_n. Let X_k and Y_k be the color classes of H_k. (The set $V(H_k) = X_k \cup Y_k$ is not necessarily all of $V(K_n)$.)
We assign an $n \times n$ matrix A_k to each graph H_k. The entry of A_k in the ith row and jth column is

$$a_{ij}^{(k)} = \begin{cases} 1 & \text{if } i \in X_k \text{ and } j \in Y_k \\ 0 & \text{otherwise} \end{cases}$$

We claim that each of the matrices A_k has rank 1. This is because all the nonzero rows of A_k are equal to the same vector, namely, the vector with 1s at positions whose indices belong to Y_k and with 0s elsewhere. (2)

問題-1-2. [線形代数の復習]

(1) 正方行列 A, B に対して $\mathrm{rank}(A \cdot B) \leq \mathrm{rank}(A)$ であることを示せ。

(2) ベクトル空間 V の基底の定義をいえ。

(3) ２次元ベクトル空間 V の任意の３つのベクトルは一次従属であることを示せ。

(4) $V, W \subset \mathbb{R}^n$ は部分ベクトル空間とする。このとき、

$$\dim(V + W) = \dim(V) + \dim(W) - \dim(V \cap W)$$

を示せ。

(5) $v_1, v_2 \cdots, v_n \in V$ を $\dim(V) = n$ のベクトルとする。このとき、これらが一次独立であれば、基底であることを示せ。

(6) A を n 次正方行列とし、$v \in \mathbb{C}^n$ とする。このとき、連立方程式 $Av = 0$ に自明でない解があることと、$\det(A) = 0$ であることは同値であることを示せ。

(7) ３つのベクトル v_1, v_2, v_3 のうちどの２つをとっても一次独立であるとき、これらは一次独立か？

(8) $d/dt : \mathbb{R}[t]_2 \to \mathbb{R}[t]_2$ を微分写像とする。このとき、線形写像 d/dt を適当な基底によって表現せよ。

(9) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ を直交行列によって対角化せよ。

(10) 2×2 正方行列で、対角化できない行列の例をあげよ。

- abstract 抽象的な (Vectors are regarded as abstract mathematical objects with particular properties.)
- amount to ということになる (The Itô stochastic integral amounts to an integral with respect to a function.) stochastic integral (確率積分)
- assign : 割り当てる (Each vector v can be assigned coordinates.)
- axiomatize 公理化する (The first reaction of mathematicians for those paradox was to ’ axiomatize’ Cantor’s intuitive set theory.)
- anyhow いずれにせよ (You should study differential geometry anyway.)
- be force to do することを強制する (The objective of the flight control system is to force the missile to achieve he steering commands.)
- bipartite graph 二部グラフ
- call A B : A のことを B とよぶ (We call those number real number.)
- claim 主張する (Newton claimed that he invented differential first)
- class: 類 (Equivalent class)
- column 列 (Column vectors)
- complete graph 完全グラフ
- condition 条件 (Necessary and sufficient condition.)
- constantly いつも、絶えず (For real number x, $x^2 \geq 0$ is constantly true.)
- contain 含む (This set contains any positive integer.)
- cover : 覆う (The family of the intervals covers the real line.)

- customary 慣例の、慣例である (It is customary in mathematics to write the equation above as follows.)
- deal with ... 扱う (In this chapter we deal with a set of linear equations.)
- define A as B : A のことを B と定義する．(We define limit of a function as follows.)
- A denote B : A は B のことを示す (V denoted a vector space.)
- disjointly : ばらばらに (共通部分なく) (These sets are embedded in the space disjointly.)
- distance : 距離 (The distance between the two points is 2.)
- edge 辺 (We call the set *edge*.)
- element : 要素、元 (Any element in the set A is also contained in B.)
- entry : 行列の各成分 (Any entry of the matrix is positive real number.)
- exclude 除外する (If several exceptions are excluded, this theorem may be true.)
- exercise 練習問題 (This is an easy exercise.)
- finitely many : 有限個の (The set contains finitely many elements.)
- for instance 例えば (For instance, the equation satisfies with the property.)
- a fortiori なおさら (Complex number satisfies it, a fortiori, so is real number.)
- fraction 分数 (Partial fraction decomposition)
- function 関数 (Continuous function.)
- graph グラフ (The graph theory.)
- indeed 実際 (Indeed, the computation is a hard work.)
- induction 帰納法 (The formula is proven by induction.)
- infinitely many : 無限個の (The linear equation admits infinitely many solutions.)
- in practice 実際には (In practice, it is difficult to prove the conjecture.)
- integer: 整数 (Integer valued-polynomial.)
- interior : 内部 (The interior of the set is an open set.)
- irrational: 無理数 (Irrational number is the complement of the rational numbers.)
- Let A be B. : A を B とする (Let $Ax = b$ be a linear equation over a field k.)
- matrix 行列 (複数形は matrices)(The matrix representation.)
- namely　つまり (Namely, this example gives an counterexample of this conjecture.)
- not necessarily とは限らない (The solutions of the equation are not necessarily positive real numbers.)
- n-tuple: n 組み (The n-tuple of real numbers constructs a vector in $\mathbb{R}^n$.)
- observe 観察する (よく見る)(Observing this formula, we recall an identity of trigonometric functions.)
- odd : 奇数の (Odd number.)
- often しばしば (We often omit the notation.)
- pair : 組み (The pair is called a twin prime.)

- parenthesis (まる) かっこ (We need start from the innermost calculation surrounded by parentheses.)
- property 性質 (Property X is understood as follows.)
- rank ランク (Rank formula.)
- rational : 有理数 (A is a rational.)
- rectangle: 長方形 (A rectangle is a quadrilateral with four right angles)
- respectively それぞれ (The two angles are 90° and 60° respectively.)
- restrict attention to .... に注意を制限する (The readers should restrict attention to the understanding of this formula.)
- row: 行 (This vector is the first row of the matrix.)
- set : 集合 (The set contains at least one element.)
- A so that B : B となるような A(There exists functions $f_1(t)$ and $f_2(t)$ so that $f(t) = f_1(t) + f_2(t)$, where $f_1(t)$ and $f_2(t)$ are an odd function and an even function respectively.)
- square : 正方形 (The diagonals of any square are equal.)
- A such that B : B を満たす A(There exists an integer n such that $x = 2n + 1$.)
- suppose: 仮定する．(Suppose that the set is not empty.)
- take into account ... を考慮する (If the assumption is taken into account, the proposition is false.)
- theorem : 定理 (Fermat's Last Theorem.)
- There exists A: A は存在する (There exists an integer n such that for any real number r with $r > n$.)
- tile : 敷き詰める (Tiling the plane with triangles.)
- vector ベクトル (The vector space is a set.)
- vector space V over K K 上のベクトル空間 V(Let V be a vector space over K.)
- verify 確認する (We need verify that the solution satisfies the equation.)
- vertex 頂点 (A rectangle has four vertices.)
- where A, ただし A であるとする．(The number i is a complex number, where i is the imaginary unit.)
- with A : A の条件のある (The equation $at + b = 0$ with a even.)

東大の川東先生の「セミナーの準備のしかたについて」
(省略)Web 上の川東先生のページを見ること。

ホームページ：**http://www.math.tsukuba.ac.jp/~tange/jugyo/18/gai.html**
blog：(**http://motochans.blogspot.jp/**)
Twitter：**BasicMathIIB**
相談、質問などいつでも承ります．アドレスはプリント 1 ページ目上部.