NON-EXISTENCE THEOREMS ON INFINITE ORDER CORKS

MOTOO TANGE

ABSTRACT. Suppose that X, X' are simply connected closed exotic 4-manifolds. It is well-known that X' is obtained by an order 2 cork twist of X. We show that in the case of infinite order cork, this existence theorem does not always hold.

1. Introduction

1.1. **A fact for cork twist.** In smooth 4-manifolds the following existence theorem of a cork is well-known.

Fact 1.1 ([9],[4]). *Let X, X' be simply-connected closed exotic smooth 4-manifolds. Then there exists a contractible 4-manifold C in X such that $X' = (X - C) \cup_\tau C$ and $\tau^2 = id$.*

Furthermore, as such a manifold C we can take a Stein manifold [1]. 'Exotic' means that manifolds are homeomorphic but non-diffeomorphic each other. The manifold obtained by removing a submanifold $Y \subset X$ with embedding i and regluing Y via τ is denoted by $X(i, Y, \tau)$. We may omit the embedding map i in the notation, if the map is understood in that context. Here we call such a surgery simply *twist*. Hence, cork means a *localization* of exotic structure.

1.2. **Motivation and results.** As Fact 1.1 mentioning, any exotic two 4-manifolds X and X' have an involutive relationship with respect to a cork twist. What we issue is the point of whether the existence holds for an infinite family. In this paper we give a negative answer (Main theorem 1) for this question. In the local situation we have a natural question of whether a (generalized) cork twist is a result of an inner cork or not. We give a negative answer (Main theorem 2) for this question as well.

1.3. **Finite, infinite order cork, and Main theorem 1.** Let $(\mathcal{C}, \tau)$ be a pair of a smooth manifold $\mathcal{C}$ and a boundary diffeomorphism $\tau : \partial\mathcal{C} \to \partial\mathcal{C}$. If τ extends to a homeomorphism on $\mathcal{C}$ but cannot extend to any diffeomorphism on $\mathcal{C}$, then τ is called *non-trivial* (otherwise *trivial*). If $\mathcal{C}$

Date: September 26, 2016.
1991 *Mathematics Subject Classification.* 57R55, 57R65.
Key words and phrases. Infinite order cork.
The author was partially supported by JSPS KAKENHI Grant Number 26800031.

is a contractible and τ is non-trivial, then the pair $(\mathcal{C}, \tau)$ is called a *cork*. Freedman's result [6] says that $\mathcal{C}$ is a contractible and τ cannot extend to any diffeomorphism on $\mathcal{C}$, then $(\mathcal{C}, \tau)$ is a cork. When we replace the contractible condition with the non-contractible one, we call $(\mathcal{C}, \tau)$ a *non-contractible cork*. Then the map τ is called *cork map* or *non-contractible cork map*. The *order* of a cork (or non-contractible cork) is the minimal positive number of n that τ^n can extend to a diffeomorphism on $\mathcal{C}$.

The author in [14] illustrates an example of a non-contractible cork. Recently, by the author [13] and Auckly, Kim, Melvin, and Ruberman [2] finite order corks are found. Right after the discoveries, Gompf in [7] found infinite order corks.

Theorem 1.2 ([7])**.** *Suppose that K_n is the $2n$-twist knot. Then there exists an infinite order cork (C, f) satisfying $X_{K_n} = X(C, f^n)$.*

Here the 4-manifold X need have 2 vanishing cycles isotopic to the meridian of the knot-surgery. At the point that (C, f) produces Fintushel-Stern's knot-surgeries, this cork is very exciting object.

We prove the following non-existence theorem on infinite order cork. Here we denote by $\mathbb{F}$ the order 2 field $\mathbb{Z}/2\mathbb{Z}$.

Main theorem 1. *Suppose that $\{X_n\}$ is a $\mathbb{Z}$-family of exotic oriented closed 4-manifolds with $b_2^+ > 1$ giving infinite OS-invariants with $\mathbb{F}$-coefficient. Then, there exists no infinite order cork (C, τ) such that $\{X_n\} = \{X(C, \tau^n)\}$.*

This theorem would be true even if one replaces OS-invariant with Seiberg-Witten invariant with $\mathbb{F}$-coefficient, because of the equivalence of the OS-invariant and the Seiberg-Witten invariant. This equivalence for 4-manifolds is still open now.

For a closed spin^c 4-manifold $(X, \mathfrak{s})$ the OS-invariant $\Phi_{X,\mathfrak{s}}$ is a smooth 4-manifold invariant

$$\Phi_{X,\mathfrak{s}} \in \mathbb{F}.$$

Then we have a polynomial

$$\sum_{\mathfrak{s} \in \mathrm{Spin}^c(X)} \Phi_{X,\mathfrak{s}} \cdot e^{PD[c_1(\mathfrak{s})]} =: \Phi_X.$$

We call the polynomial *OS-invariant*. As an application, the following corollary holds.

Corollary 1.3. *Suppose that T_n is the $(2, 2n+1)$-torus knot. Then for any integer m with $m \geq 2$ the family $\{E(m)_{T_n}\}$ cannot be constructed by twisting an infinite order cork.*

Compared this corollary with Theorem 1.2, we know that the two situations are contrasting. The $\mathbb{F}$-reductions of $\{\Delta_{T_n}(t)\}$ are infinite, i.e.,

$$\#\left\{\sum_{k=1}^{n} t^k | n \in \mathbb{Z}\right\} = \infty,$$

while the $\mathbb{F}$-reductions of $\{\Delta_{K_n}(t)\}$ are finite, precisely saying

$$\#\{1, t-1+t^{-1}\} = 2.$$

Here we denote by $\Delta_K(t)$ the Alexander polynomial of K. Depending on the knot, the existence of infinite order cork for Fintushel-Stern's knot-surgery changes.

This theorem means that the OS-invariants with $\mathbb{F}$-coefficient of 4-manifolds obtained from a single (finite or infinite order) cork are finite variations. Thus, immediately, we have the following corollary:

Corollary 1.4. *Let $\mathcal{C}$ be a contractible 4-manifold. Let r be a rank of $HF^-(\partial\mathcal{C},\mathbb{F})/(U=0)$. If $\mathcal{C}$ admits a G-cork with a G-effective embedding with distinct $\mathbb{F}$-coefficient OS-invariants, then $|G|\le\prod_{k=0}^{r-1}(2^r-2^k)$ holds.*

Question 1.5. *Let $\{X_n\}$ be an exotic family of 4-manifolds (e.g., with distinct $\mathbb{Z}$-coefficients OS-invariants). Suppose that $\{X_n\}$ have finite $\mathbb{F}$-coefficients OS-invariants. Then does there exist an infinite order cork $(\mathcal{C},\tau)$ which produces $\{X_n\}$?*

A remaining question is a characterization of the finite family which is produced by a finite order cork $(\mathcal{C},\tau)$.

Question 1.6. *Let $\{X_k|k=0,\cdots,n-1\}$ be a finite family of exotic 4-manifolds. When does there exist an order n cork $(\mathcal{C},\tau)$ such that the family is obtained by cork twists of $(\mathcal{C},\tau)$.*

Let $\mathcal{C}$ be a 4-manifold and $\mathcal{D}$ a contractible submanifold of $\mathcal{C}$ with $\dim\mathcal{C} = \dim\mathcal{D}$ and with $\partial\mathcal{D}$ smoothly embedded in the interior of $\mathcal{C}$. Let i be the identity map $\partial\mathcal{C}\to\partial\mathcal{C}$. Thus, $(\mathcal{C},i)$ is a trivial twist. Let g be a boundary diffeomorphism of $\mathcal{D}$. Suppose that there exists a diffeomorphism F from the twist $\mathcal{C}(\mathcal{D},g)$ to $\mathcal{C}$. Then i induces a diffeomorphism $\partial\mathcal{C}\to\partial\mathcal{C}(\mathcal{D},g)$. We define the composition $F^{-1}|_{\partial\mathcal{C}}\circ i$ by j. We call $(\mathcal{C},j)$ (or $(\mathcal{D},g)$) *an induced twist* of $(\mathcal{D},g)$ (or core twist of $(\mathcal{C},f)$ respectively).

$$\begin{array}{ccc} \mathcal{C} & \xrightarrow{F} & \mathcal{C}(\mathcal{D},g) \\ & & \Big\uparrow \text{inclusion} \\ \partial\mathcal{C} & \xrightarrow{i} & \partial\mathcal{C}(\mathcal{D},g) \end{array}$$

It is already not clear whether $(\mathcal{C},j)$ is trivial. Then we denote it by

$$(\mathcal{D},g)\subset(\mathcal{C},f).$$

Definition 1.7 (Core cork and induced cork.)**.** *Suppose that $(\mathcal{D},g)\subset(\mathcal{C},f)$. If $(\mathcal{C},f)$ is a cork, then the twist $(\mathcal{D},g)$ is also a cork. In this case we call $(\mathcal{D},g)$ a core cork of $(\mathcal{C},f)$.*

For the case where $(\mathcal{C},f)$ or $(\mathcal{D},g)$ is a plug or non-contractible cork we use the same terminology $\subset$ in the similar situation. Even if a cork twist $(\mathcal{D},g)$ induces a twist $(\mathcal{C},f)$, then $(\mathcal{C},f)$ is not always a cork (or non-contractible cork) twist. The orders of $(\mathcal{C},f)$ and $(\mathcal{D},g)$ do not always agree with each other. For example, in [14] the author proved cork-ness of $(D_{n,m},\tau^D_{n,m})$ (order n) by using an induced twist $(D_{n,m},\tau^D_{n,m})\subset(C(m),\tau(m))$ and what $(C(m),\tau(m))$ is an order 2 Stein cork.

Furthermore, we consider the following concept for a family version of core (and indued) cork.

Definition 1.8 (Core G-cork, induced H-cork)**.** *Let $(\mathcal{D},G)$ be a G-cork and a submanifold in a 4-manifold $\mathcal{C}$ with boundary and $\partial\mathcal{D}\subset\mathcal{C}$ smoothly embedding. Assume that $\mathcal{C}-\mathcal{D}$ is not diffeomorphic to a cylinder of $\partial\mathcal{C}$. If any $g\in G$ gives an induced twist $(\mathcal{D},g)\subset(\mathcal{C},h)$ and the correspondence $g\mapsto h$ produces an isomorphism*

$$G\overset{\cong}{\to}H\subset \mathit{Diff}(\partial\mathcal{C})$$

into a subgroup H, then $(\mathcal{D},G)$ is called a core G-cork of $(\mathcal{C},H)$ and $(\mathcal{C},H)$ is called an induced H-cork of $(\mathcal{D},G)$. Then we denote it by

$$(\mathcal{D},G)\subset(\mathcal{C},H).$$

In [13], we prove the $\mathbb{Z}_2$-cork $(C(1),\{\tau(1)\})$ contains a core $\mathbb{Z}_2$-cork

$$(C_{2,1},\{\tau^C_{2,1}\})\subset(C(1),\{\tau(1)\}),$$

because, $\partial C_{2,1}$ and $\partial C(1)$ are not diffeomorphic homology spheres because of SnapPea computation. Therefore, $C(1)-C_{2,1}$ is not diffeomorphic to the cylinder. It is an open question whether $\partial C_{2,m}\not\cong\partial C(m)$ or not for any m. The motivation of core cork is to replace a cork or non-contractible cork twist with a new (or possible 'universal') reasonable cork.

To find a cork in a wider situation we would like to search a cork in a non-contractible cork. As an application of Main theorem 1 we show the following theorem.

Main theorem 2. *There exists a non-contractible $\mathbb{Z}$-cork $(\mathcal{P},\mathbb{Z})$ such that $(\mathcal{P},\mathbb{Z})$ never contain any core $\mathbb{Z}$-cork.*

We give a natural question:

Question 1.9. *Let H be a finite group. Does any non-contractible H-cork contain a core G-cork $\mathcal{D}$ with $(\mathcal{D},G)\subset(\mathcal{C},H)$?*

1.4. **A Stein plug (Q,ϕ) with $b_2(Q)=1$ changing any crossing of Fintushel-Stern's knot-surgery.** Let (P,φ) be a plug with $b_2=2$ which is defined in [14]. The last assertion in this paper is that it is not a plug with the minimal b_2 which gives rise to any crossing change of Fintushel-Stern's knot-surgery. Let Q be a 4-manifold obtained by attaching a 2-handle along 5_2 with 0-framing. The 4-manifold Q is a submanifold in P naturally. See the first handle diagram of Q in Figure 4. Hence, we have $\partial Q\cong S^3_0(5_2)$. Then we prove the following:

Proposition 1.10. *There exists a diffeomorphism $\phi:\partial Q\to\partial Q$ such that (Q,ϕ) is a Stein core $\mathbb{Z}$-plug of (P,φ).*

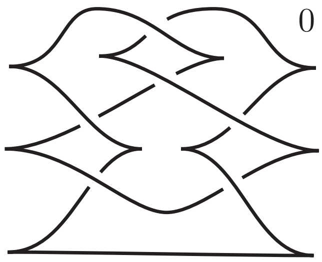

FIGURE 1. A Stein structure on Q.

A Stein structure on Q is presented in FIGURE 1. This $\mathbb{Z}$-plug (Q, ϕ) produces infinitely many exotic Fintushel-Stern's knot-surgeries. This means that the action on the Heegaard Floer homology should admit infinite order.

Let $\mathfrak{s}_k$ be a spin^c structure with $\langle c_1(\mathfrak{s}_k), h\rangle = 2k$, where h is a generator in $H_2(\partial Q)$.

The Heegaard Floer homology of ∂Q is as follows:

$$HF^-(\partial Q, \mathfrak{s}_k) \cong \begin{cases} T^-_{(-\frac{5}{2})} \oplus T^-_{(-\frac{7}{2})} \oplus \mathbb{F}_{(-\frac{5}{2})} & k = 0 \\ \mathbb{F}[U]/(U^k - 1) & k \neq 0. \end{cases}$$

This computation will be done in Section 3.2. The action on the Heegaard Floer homology of ∂Q should be effective. This fact is contrast to Main Theorem 1. To investigate the mechanism that any crossing change of Fintushel-Stern's knot surgery changes the differential structures in terms of Heegaard Floer homology might become a help to understand exotic structures on 4-manifolds.

ACKNOWLEDGEMENTS

The main results were produced by an argument with Robert Gompf who gave me suggestions. The author thanks him for his helps for this earlier manuscript of this paper. Furthermore, the author is deeply grateful with arguments with Kouichi Yasui in the Hiroshima University Topology-Geometry seminar.

2. PRELIMINARIES AND PROOFS OF MAIN THEOREM 1 AND 2

2.1. **Knot-surgery.** Let K be a knot in S^3. Let X be a 4-manifold with a square zero embedded torus T. Then the performance

$$X_K = [X - \nu(T)] \cup [(S^3 - \nu(K)) \times S^1]$$

is a *(Fintushel-Stern's) knot-surgery* along K. The gluing map is indicated in [5]. The notation $\nu(\cdot)$ stands for an open neighborhood of a submanifold.

T. Mark in [8] proved the knot-surgery formula of ($\mathbb{F}$-coefficient) Ozsváth-Szabó's 4-manifold invariant.

$$\Phi_{X_K} = \Phi_X \cdot \Delta_K(t). \quad (1)$$

This is an Ozsváth-Szabó's invariant counterpart of the Seiberg-Witten formula of Fintushel-Stern's knot-surgery in [5]. Here we have to notice Sunukjian's result [12] that the Alexander polynomial distinguishes smooth structures of Fintushel-Stern's knot surgeries.

2.2. **Proof of Main theorem 1.** Suppose that $\{X_n\}$ is an exotic $\mathbb{Z}$-family of closed 4-manifolds with $b^+(X) > 1$ having infinite OS-invariants with $\mathbb{F}$-coefficient and these are produced by cork twists by an infinite order cork $(\mathcal{C}, \tau)$. If X_n are not closed, then the same argument works by the relative OS-invariant.

By permuting the order of X_n, we have $X(\mathcal{C}, \tau^n) = X_n$. The group $\langle\tau\rangle \cong \mathbb{Z}$ acts on $\partial\mathcal{C}$ and the action induces a homomorphism on $HF^-(\partial\mathcal{C})$.

The induced isomorphism τ_* on $HF^-(\partial\mathcal{C})$ keeps the absolute grading. The grading shift of the action is calculated from the Euler number and the signature of the cylinder $I \times \partial\mathcal{C}$. These invariants of the cylinder are all zero. $HF^-_d(\partial\mathcal{C})$ with a fixed grading d is a finite abelian group which is isomorphic to $\mathbb{F}$ for sufficient small d's. Hence there exists a positive integer m such that τ_*^m is the identity.

Here we consider X_n as the gluing of three 4-manifolds $\{\mathcal{C}, M, \tilde{V}\}$, where $X_n = [\mathcal{C} \cup_{\tau^n} M] \cup_N \tilde{V}$ and M is a cobordism $\partial\mathcal{C} \to \partial\tilde{V}$. Furthermore, we assume that N is an admissible cutting of X_n (defined in [11]). The existences of these cuttings N are guaranteed in [11]. Deleting two 4-balls in the interior in X_n, we give the composition W_n of three cobordisms:

$$W_n : S^3 \xrightarrow{\mathcal{C}_0} \partial\mathcal{C} \xrightarrow{M} N \xrightarrow{V} S^3,$$

where actually the cobordism W_n is twisted on $\partial\mathcal{C}$ by action τ^n and V is $\tilde{V}$ with a 4-ball deleted. Here the mixed invariant on W_n becomes as follows:

$$F^{\mathrm{mix}}_{W_n,\mathfrak{s}_0} = F^+_{V,\mathfrak{s}_3} \circ F^-_{M,\mathfrak{s}_2} \circ \tau_*^n \circ F^-_{\mathcal{C}_0,\mathfrak{s}_1} : HF^-(S^3) \to HF^+(S^3),$$

where $\mathfrak{s}|_{W_n} = \mathfrak{s}_0$, $\mathfrak{s}|_{\mathcal{C}_0} = \mathfrak{s}_1$, $\mathfrak{s}_0|_M = \mathfrak{s}_2$ and $\mathfrak{s}_0|_V = \mathfrak{s}_3$. Recall the OS-invariant $\Phi_{X_n,\mathfrak{s}} \in \mathbb{F}$ is defined by $F^{\mathrm{mix}}_{W_n,\mathfrak{s}}(U^d \cdot \Theta^-) = \Phi_{X_n,\mathfrak{s}} \cdot \Theta^+$, where $d = (c_1^2(\mathfrak{s}) - 2\chi(X_n) - 3\sigma(X_n))/4$. Since $\{\tau_*^n | n \in \mathbb{Z}\}$ has a finite variation, the mixed invariant $F^{\mathrm{mix}}_{W_n,\mathfrak{s}_0}$ is also finite variations with respect to n. Thus the sets $\{\Phi_{X_n,\mathfrak{s}} | \mathfrak{s} \in \mathrm{Spin}^c(X_n)\}$ are also finite variations only with respect to n. This contradicts that $\{X_n\}$ has infinite OS-invariants with $\mathbb{F}$-coefficient. □

By a corollary we have the following:

Corollary 2.1. *Regardless of the order of the cork, the variations of $\mathbb{F}$-coefficient OS-invariants by a single cork are at most finite.*

In the case of the $\mathbb{Z}$-coefficient invariant, the variations are not always finite as a Gompf's example in [7] implies.

Proof of Corollary 1.3. Let T_n be the $(2, 2n+1)$-torus knot. Due to the OS-invariant formula (1) of $E(m)_{T_n}$ with the $\mathbb{F}$-coefficient, we have

$$\Phi_{E(m)_{T_n}} = (t - t^{-1})^{m-2}(t^n - t^{n-1} + \cdots - t^{-n+1} + t^{-n}) \bmod 2.$$

These give infinite OS-invariants. From Main theorem 1, the family $\{E(m)_{T_n} | n \in \mathbb{Z}\}$ never be produced by cork twists of an infinite order cork. □

This proof means that for a family $\{\mathcal{K}_n\}$ of knots, if $E(m)_{\mathcal{K}_n}$ is constructed by an infinite order cork, then $\#\{\Delta_{\mathcal{K}_n} \bmod 2\} < \infty$. In fact $2n$-twist knot K_n in [7] is $\{\Delta_{K_n}(t) \bmod 2\} = \{1, t-1+t^{-1}\}$.

Proof of Corollary 1.4. If a G-cork twist gives distinct $\mathbb{F}$-coefficient OS-invariants, then the action is effective on $HF^-(\partial\mathcal{C},\mathbb{F})/(U=0) = \mathbb{F}^r$. We note that the action is U-equivariant. The induced action become an invertible linear action on $\mathbb{F}^r$. Hence, we obtain $G \subset GL(r,\mathbb{F})$. Then we have $|G| \leq |GL(r,\mathbb{F})| = \prod_{k=0}^{r-1}(2^r - 2^k)$. □

2.3. **Proof of Main theorem 2.** Let (P,φ) be the plug defined in [14]. Namely, P and φ are described in Figure 2 and 3 respectively.

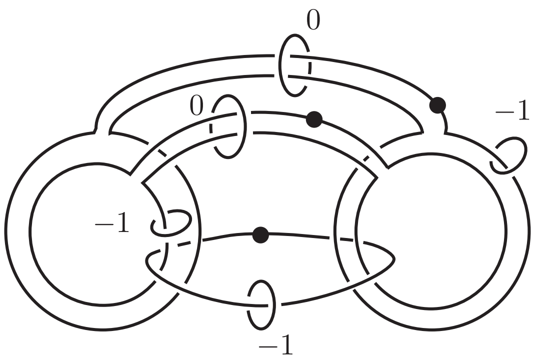

Figure 2. P.

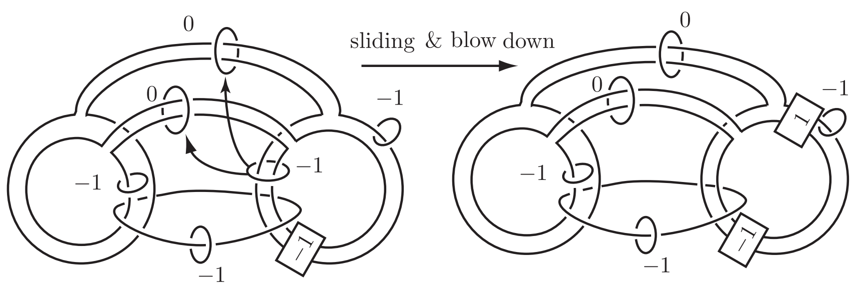

Figure 3. The diffeomorphism φ.

Taking $\psi = \varphi^2$, we obtain a non-contractible cork (P,ψ) by [14]. Lemma 3.2 in [14] says that ψ induces the trivial map on the homology group.

Since (P,φ) changes any crossing for Fintushel-Stern's knot-surgery, there exists an embedding $P \hookrightarrow E(2)$ such that the twist obtains $E(2)(P,\psi^n) = E(2)_{T_{2n}}$ as proven in [14].

Suppose there exists a core $\mathbb{Z}$-cork $(\mathcal{D},f) \subset (P,\psi)$ which $(\mathcal{D},f^k)$ induces (P,ψ^k). Since $\mathcal{D}$ is a contractible, clearly $P - \mathcal{D}$ is not diffeomorphic to a cylinder. This setting says that this infinite order cork twist gives the following twist:

$$E(2) \rightsquigarrow E(2)(\mathcal{D},f^k) = E(2)_{T_{2k}}.$$

However, the manifolds $E(2)_{T_{2k}}$ have infinite OS-invariants with $\mathbb{F}$-coefficient. This is a contradiction on Main theorem 1. □

3. A core plug of (P,φ).

3.1. **Proof of Proposition 1.10.** The first picture (denoted by Q) in Figure 4 is P deleting an embedded disk. The diffeomorphism Figure 3 works

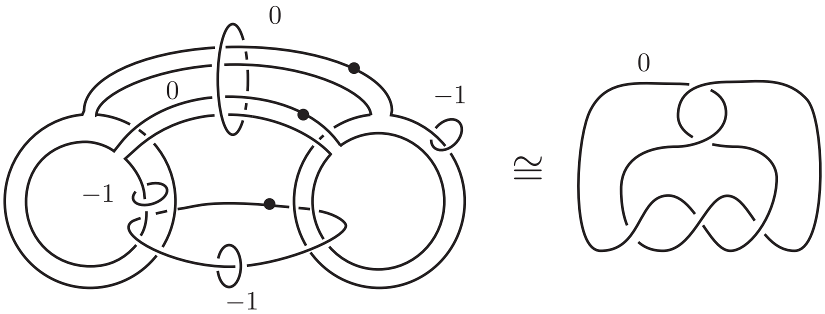

Figure 4. From the 2-handle deleted P to 0-framed 5_2

even for this submanifold Q. We denote the diffeomorphism $\partial Q \to \partial Q$ by ϕ. Thus we obtain $(Q,\phi^k) \subset (P,\varphi^k)$ for any k.

Since the twisted double $Q \cup_{\phi^k} (-Q)$ by (Q,ϕ^k) is homeomorphic to $\mathbb{C}P^2\#\overline{\mathbb{C}P^2}$ (k: odd) and $S^2\times S^2$ (k: even) by easy calculation. The diffeomorphism ϕ^{2k+1} cannot extend to a homeomorphism on Q by [3]. Thus (Q,ϕ^{2k+1}) is a plug and (Q,ϕ^{2k}) is a non-contractible cork. Hence $(Q,\{\phi^k\})$ is a core $\mathbb{Z}$-plug of $(P,\{\varphi^k\})$.

Therefore, for an unknotting number 1 knot K, there exists an embedding $Q \hookrightarrow E(2)$ such that $E(2)(Q,\phi) = E(2)_K$. Thus (Q,ϕ) is infinite order.

The handle diagram of Q can be reduced to 5_2 with framing 0. The maximal Thurston-Bennequin invariant of 5_2 is 1. Thus the manifold is Stein manifold. For example see Figure 1. □

The presentation in Figure 1 is the famous Chekanov-Eliashberg knot.

$Q\cup_\phi(-Q)$ and $Q\cup_{\phi^2}(-Q)$ is diffeomorphic to $\mathbb{C}P^2\#\overline{\mathbb{C}P^2}$ and $S^2\times S^2$ respectively.

Conjecture 3.1. *For $k \neq 1,2$ any double $Q\cup_{\phi^k}(-Q)$ is a standard 4-manifold.*

3.2. **An action ϕ_* on $HF^-(\partial Q,\mathfrak{s}_k)$.** Finally we compute the Heegaard Floer homology of ∂Q and consider the action on the homology induced by ϕ. Since 5_2 is an alternating knot, we have the following computation:

$$\widehat{HFK}(5_2,i) = \begin{cases} \mathbb{F}^2_{(0)} & i=1 \\ \mathbb{F}^3_{(-1)} & i=0 \\ \mathbb{F}^2_{(-2)} & i=-1. \end{cases}$$

Now, the Heegaard Floer homology of $-\Sigma(2,3,11)$ is as follows:

$$HF^+(S^3_1(5_2)) = T^+_{(-2)} \oplus \mathbb{F}_{(-2)}.$$

By using the surgery exact sequence in [10] among S^3, $S^3_0(5_2) = \partial Q$, and $S^3_1(5_2) = -\Sigma(2,3,11)$ we compute

$$HF^+(\partial Q, \mathfrak{s}_k) = \begin{cases} T^+_{(-\frac{1}{2})} \oplus T^+_{(-\frac{3}{2})} \oplus \mathbb{F}_{(-\frac{3}{2})} & k = 0 \\ 0 & k \neq 0. \end{cases}$$

By using the exact sequence among HF^-, HF^∞, and HF^+, we have the following computation:

$$HF^-(\partial Q, \mathfrak{s}_k) \cong \begin{cases} T^-_{(-\frac{5}{2})} \oplus T^-_{(-\frac{7}{2})} \oplus \mathbb{F}_{(-\frac{5}{2})} & k = 0 \\ \mathbb{F}[U]/(U^k - 1) & k \neq 0. \end{cases}$$

The twist ϕ induces an action on $HF^-(\partial Q)$ with spin^c structures preserving, because Spin$^c(\partial Q)$ is equivalent to Spin$^c(\partial Q \times I)$ naturally. Here we state the following question:

Question 3.2. *How does the diffeomorphism ϕ on Heegaard Floer invariants affect?*

To analyze the action would be significant to study the exotic structures that Fintushel-Stern's knot surgery gives.

References

[1] S. Akbulut, and R. Matveyev, *A convex decomposition theorem for 4-manifolds*, Internat. Math. Res. Notices 1998 no. 7, 371-381

[2] D. Auckly, H. Kim, P. Melvin, and D. Ruberman, *Equivalent corks.* arXiv:1602.07650.

[3] S. Boyer *Simply-connected 4-manifolds with a given boundary*, Amer. Math. Soc. vol.298(1986) no.1 331-357

[4] C. L. Curtis; M. H. Freedman; W. C.Hsiang; R. Stong, *A decomposition theorem for h-cobordant smooth simply-connected compact 4-manifolds*, Invent. Math. **123** (1996), no. 2, 343–348.

[5] R. Fintushel and R. Stern, *Knots, links and 4-manifolds*, Invent. Math. 134 (1998), 363-400.

[6] M. Freedman, *The topology of four-dimensional manifolds.* J. Differential Geom. 17 (1982), no. 3, 357-453.

[7] R. Gompf, *Infinite order corks*, arXiv:1603.05090

[8] T. Mark. *Knotted surfaces in 4-manifolds.* Forum Math. 25 (2013), no. 3, 597-637

[9] R. Matveyev, *A decomposition of smooth simply-connected h-cobordant 4-manifolds*, J. Differential Geom. **44** (1996), no. 3, 571–582.

[10] P. Ozsváth and Z. Szabó, *Holomorphic disks and knot invariants,* Adv. Math., 186 (2004) 58-116

[11] P. Ozsváth and Z. Szabó, *Holomorphic triangles and invariants for smooth four-manifolds,* Adv. Math., 202 (2006) 326-400

[12] *A note on knot surgery.* J. Knot Theory Ramifications 24 (2015), no. 9, 1520003, 5 pp.

[13] M Tange, *Finite order corks*, arXiv:1601.07589

[14] M Tange, *A plug with infinite order and some exotic 4-manifolds*, Jour. of Gökova Geom. & Top. **9**(2015) 1-17

Institute of Mathematics, University of Tsukuba, 1-1-1 Tennodai, Tsukuba, Ibaraki 305-8571, Japan
E-mail address: tange@math.tsukuba.ac.jp