Schröder-Bernstein property in a category of countable models

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Preliminary

- T for a complete theory
- M, N for models of T
- a, b, c $\cdots$ for elements
- x for a variable
- p for a type
- for a formula

Schröder-Bernstein property

Def.1.

T has the Shröder-Bernstein property(SB)
if any models M,N of T are isomorphic
whenever they are elementary bi-embeddable.

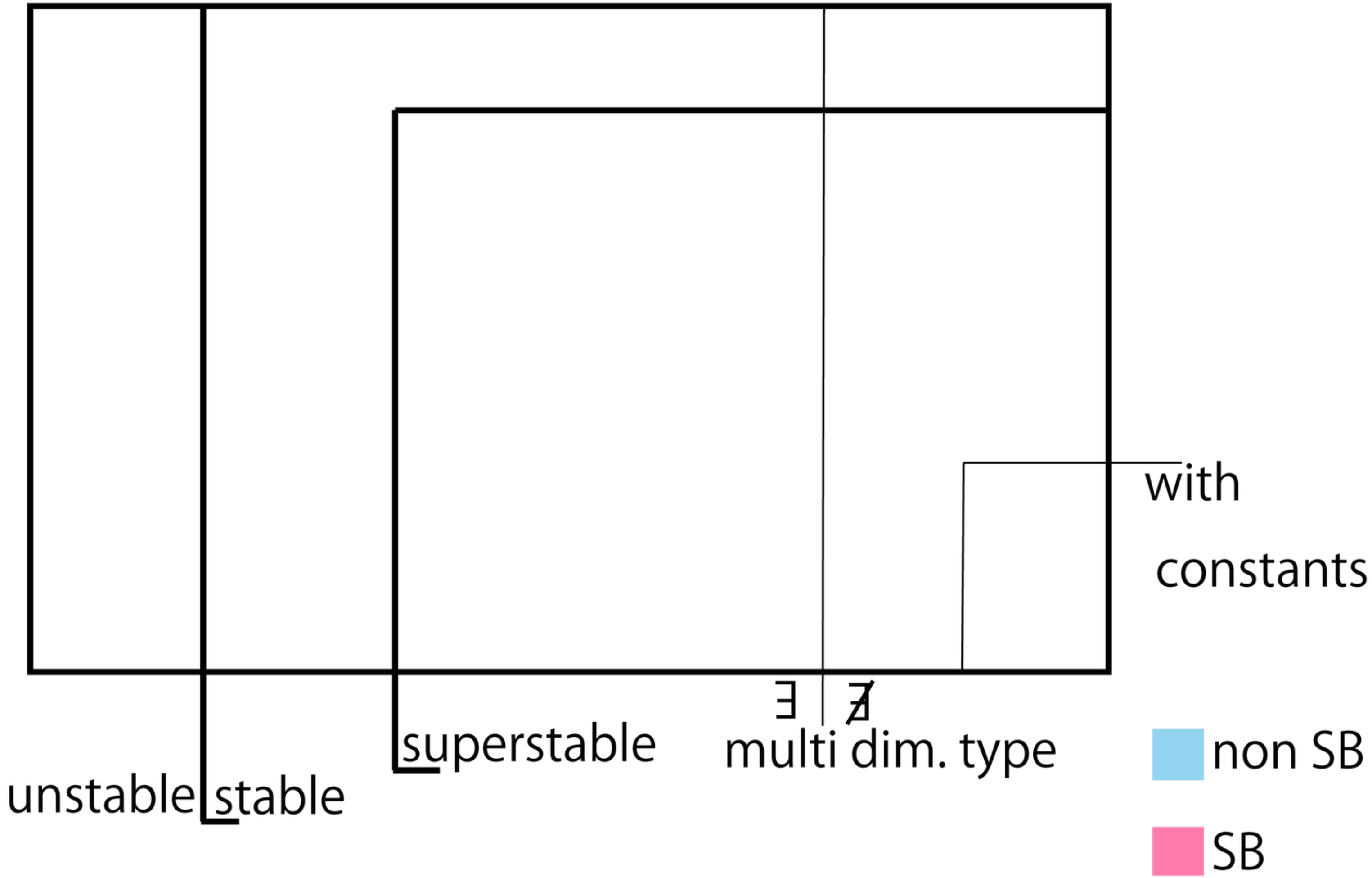
$$M \prec N, N \prec M$$

$$\downarrow$$

$$M \cong N$$

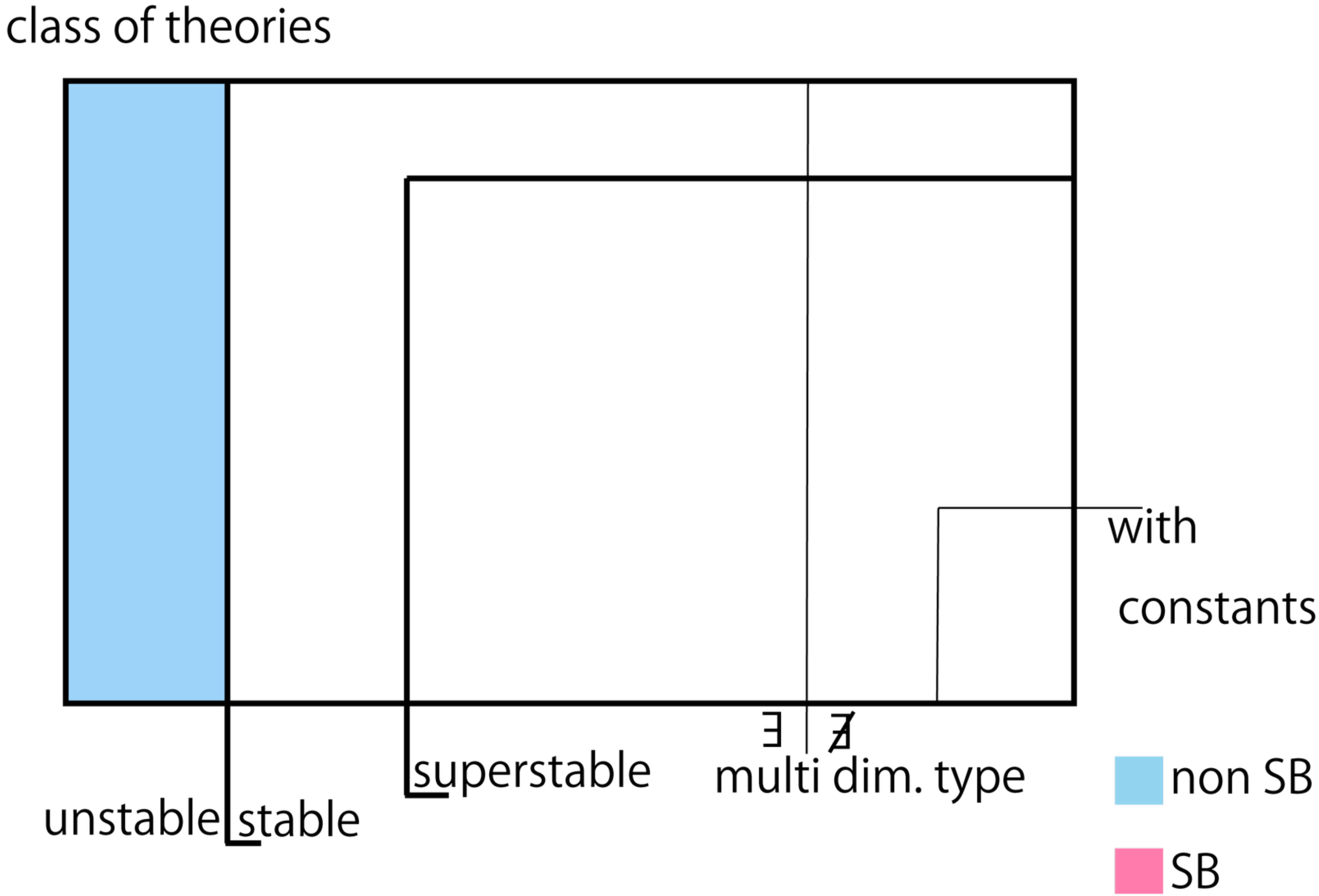
Background

class of theories



Thm.2.(S. Shelah)

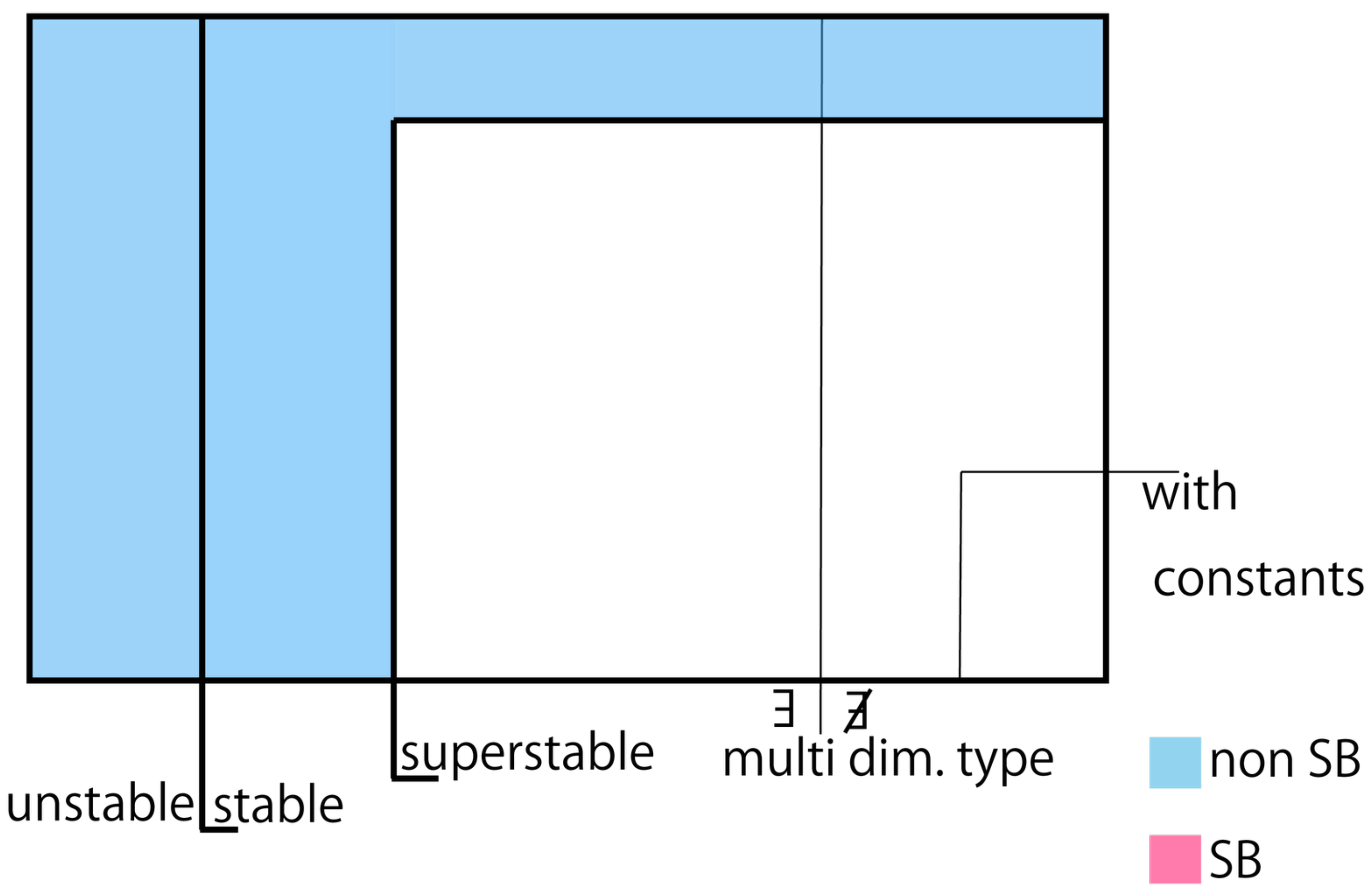
Unstable theories do not have SB.



Thm.3.(L. Harrington,2007)

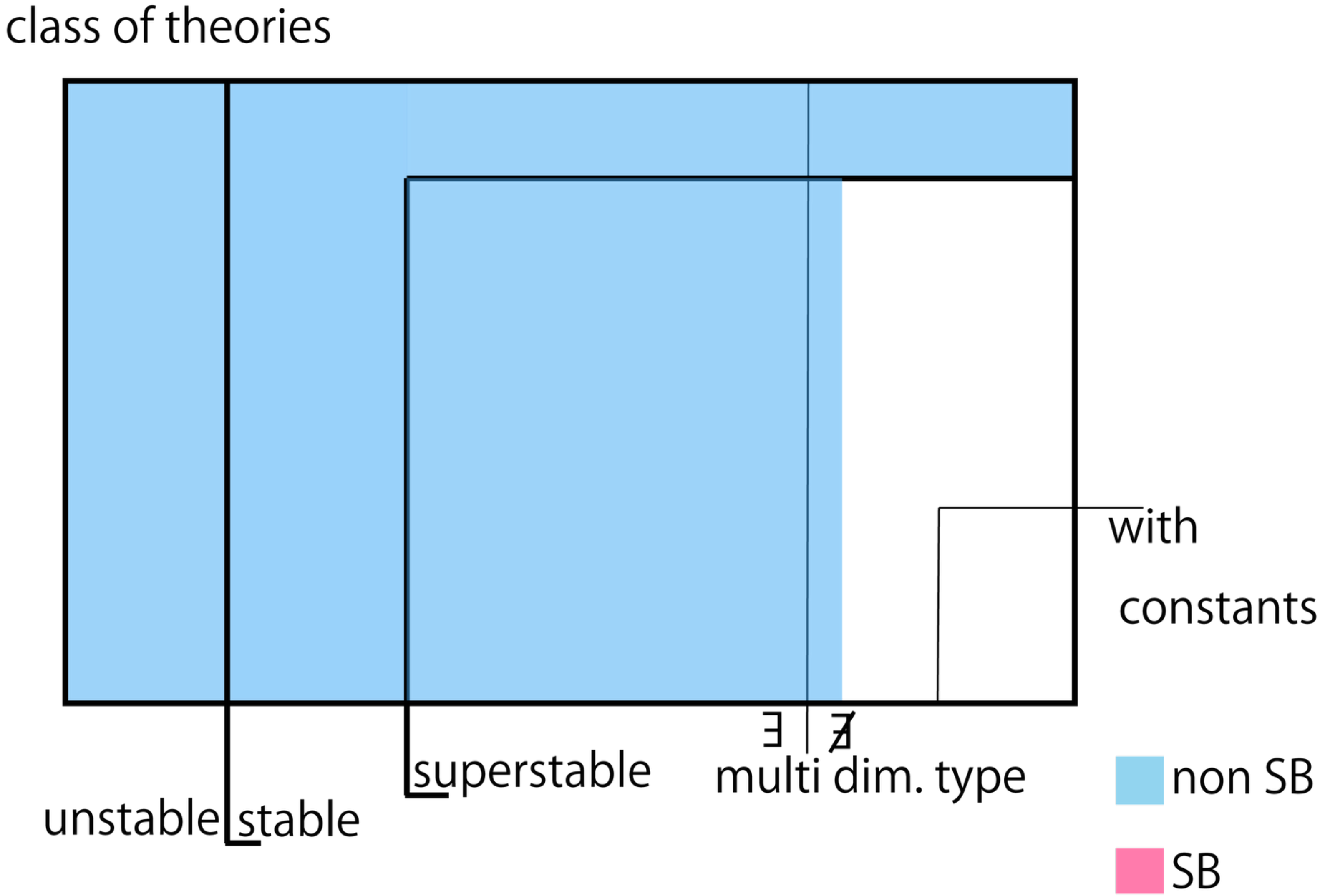
Strictly stable theories do not have SB.

class of theories



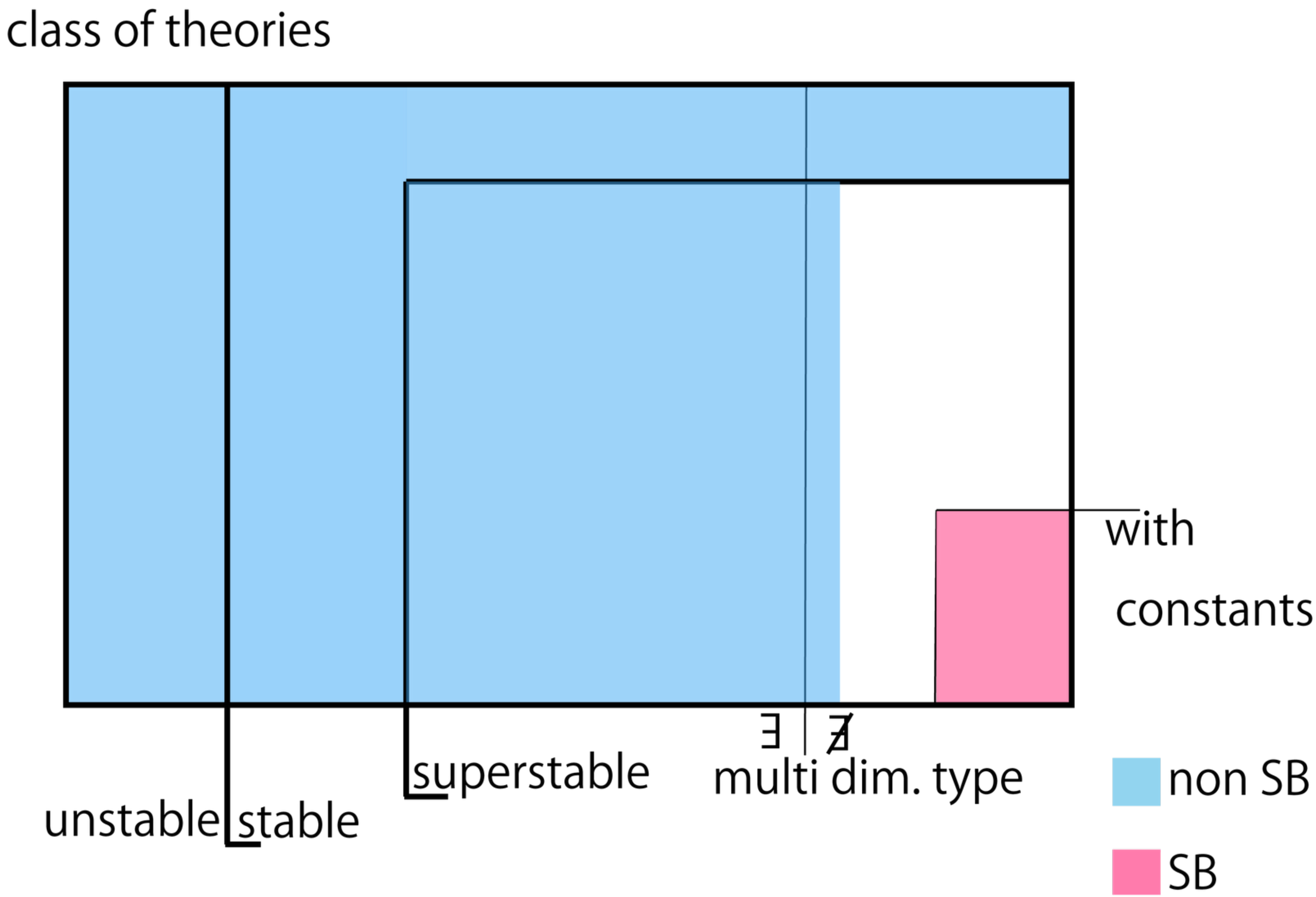
Thm.4.(J. Goodrick,2007)

Superstable theories with a nomadic type do not have SB.



Thm.5.(J. Goodrick and M. C. Lastowski,2012)

Superstable theories without nomadic types have SB after adding a-model as constants.



Cardinality of counter examples

- unstable

  at least $2^{\aleph_0}$
- strictly stable

  at least $\aleph_1$
- superstable with a nomadic type

  at least $\aleph_\omega$

Cardinality of counter examples

Q.

Can we make a countable example for non SB?

Cardinality of counter examples

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Can we make a countable example for non SB?

Suppose that T is a countable theory.

Schröder-Bernstein property for countable models

Def.6.

T has the Shröder-Bernstein property for countable models(ctbl SB) if any countable models M,N of T are isomorphic whenever they are elementary bi-embeddable.

Schröder-Bernstein property for countable models

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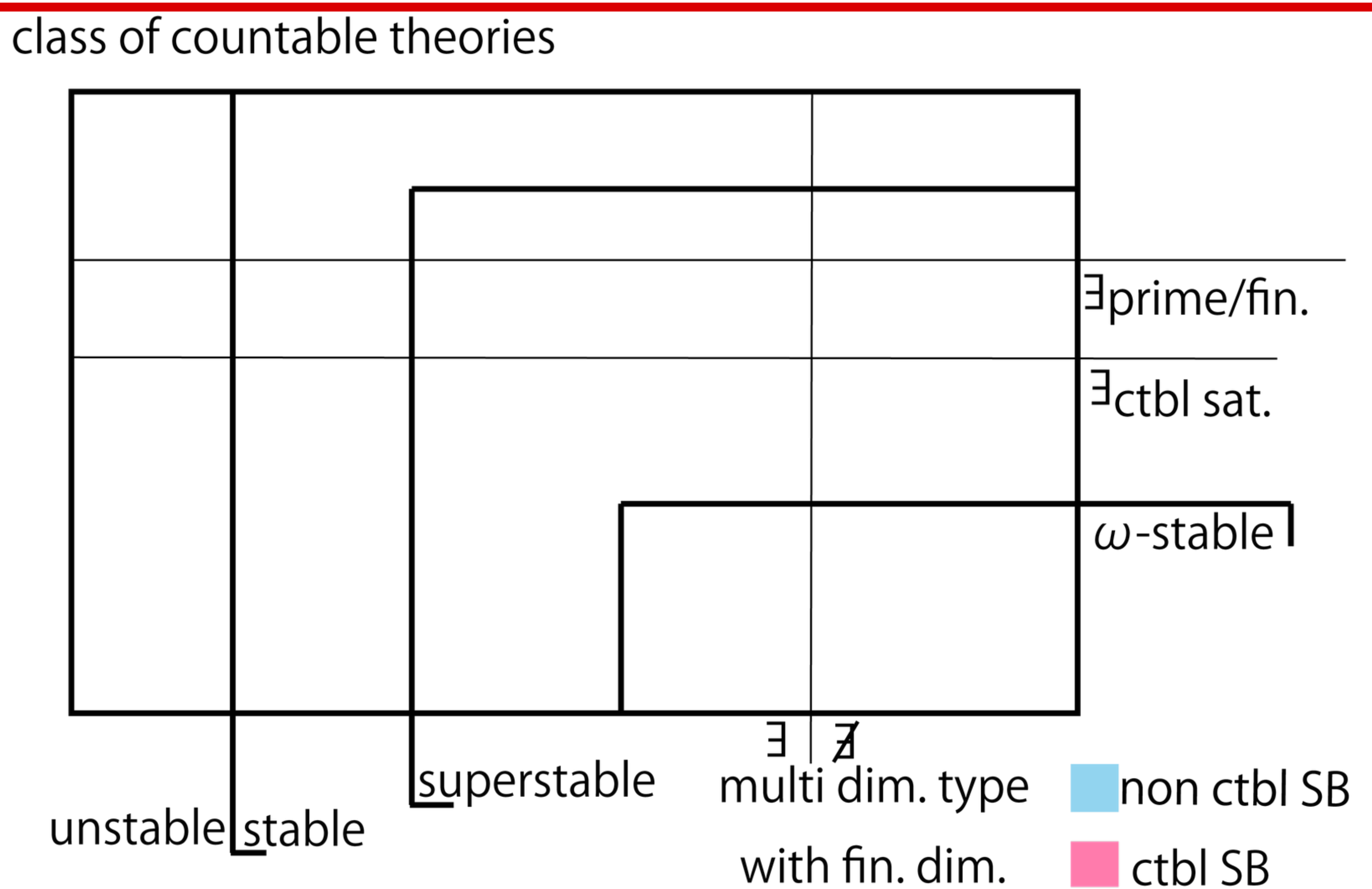
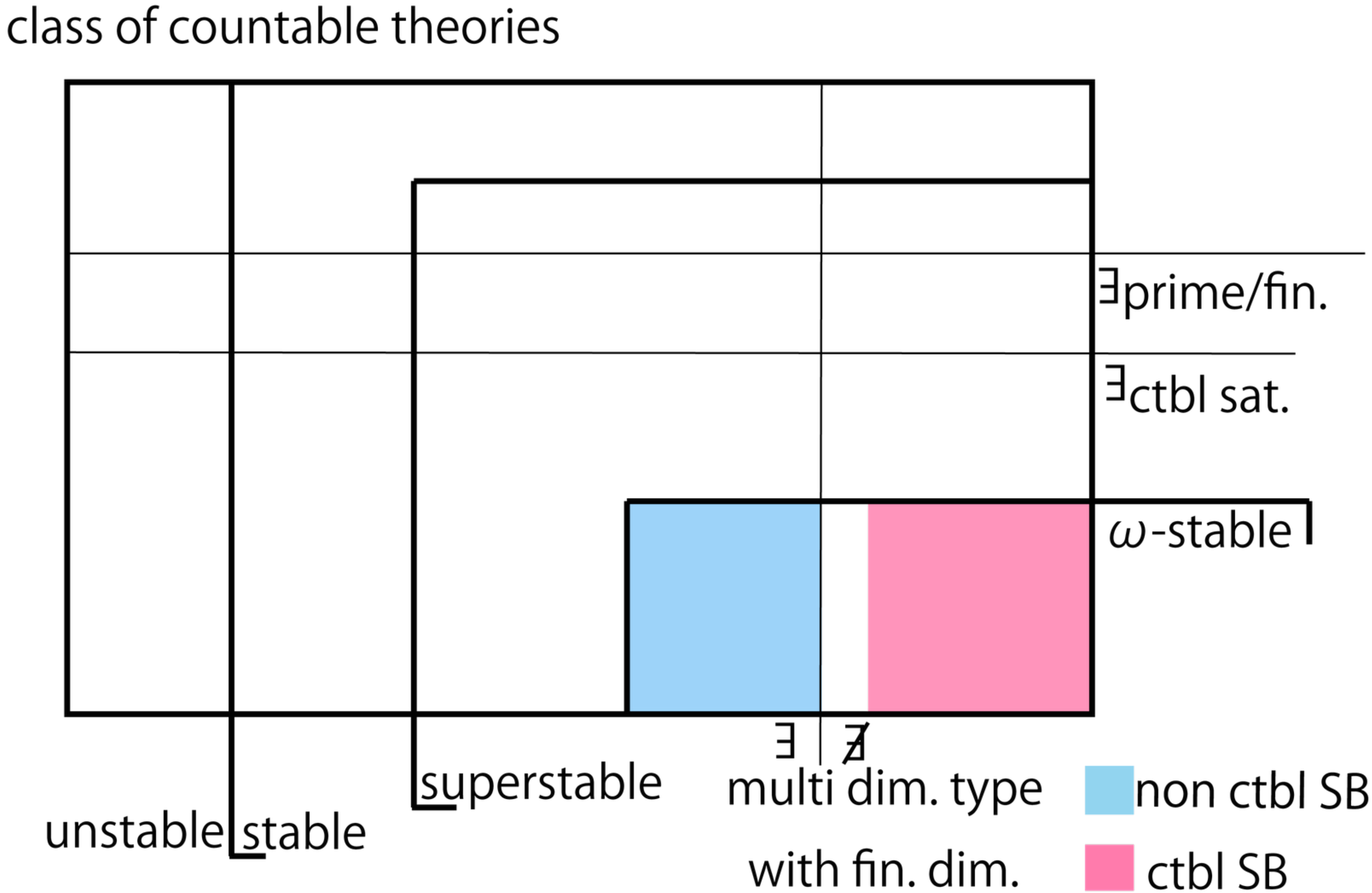
Clearly, a ω-categorical theory has ctbl SB.

Background and Results



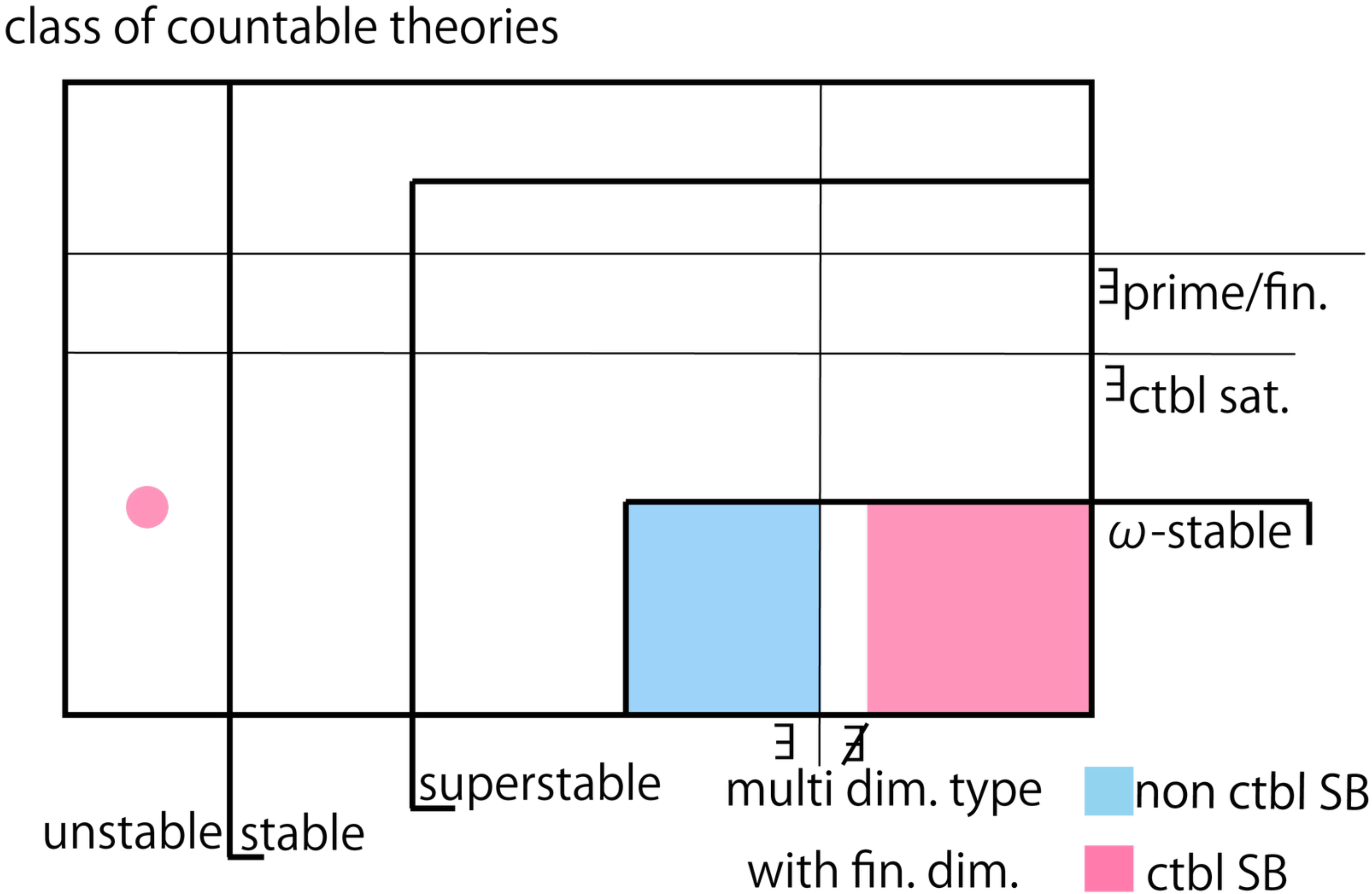
Thm.7.(T. A. Nurmagambetov,1989)

There is a characterization of ctbl SB in ω-stable case.



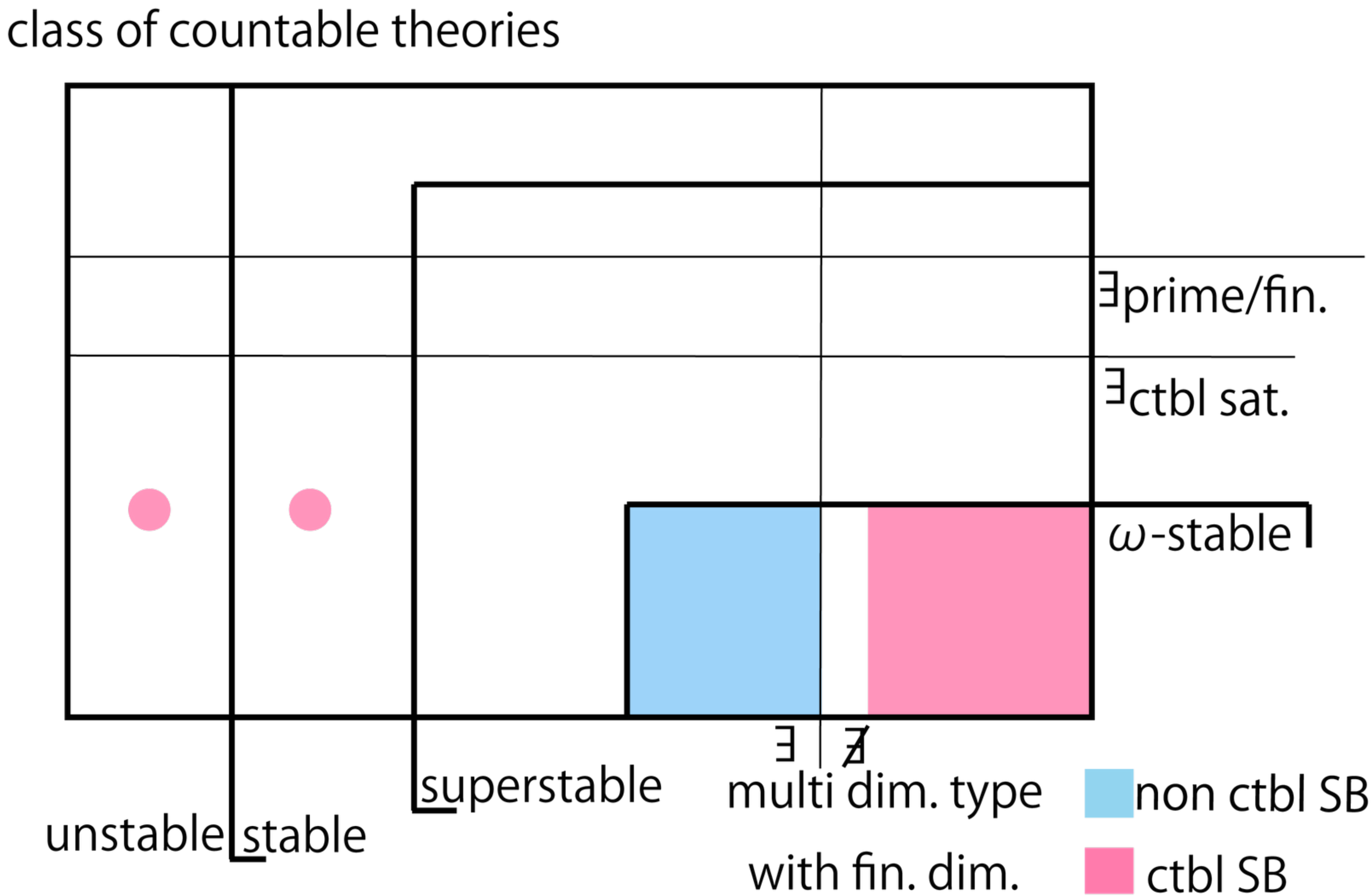
Ex.8.

The theory of a dense linear order has ctbl SB.



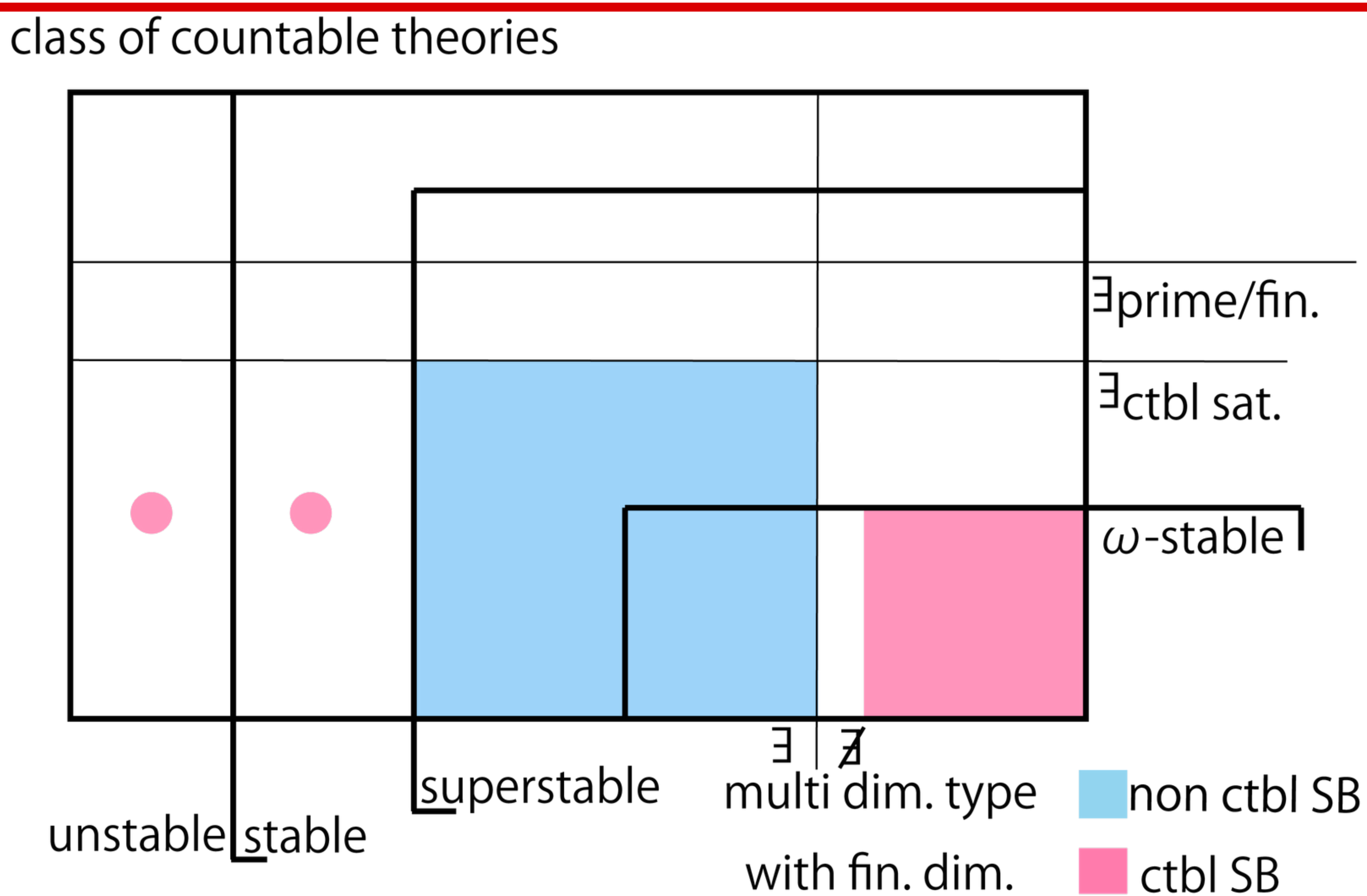
Ex.9.

There is a strictly stable ω-categorical theory.



Prop.10.(A.Tsuboi)

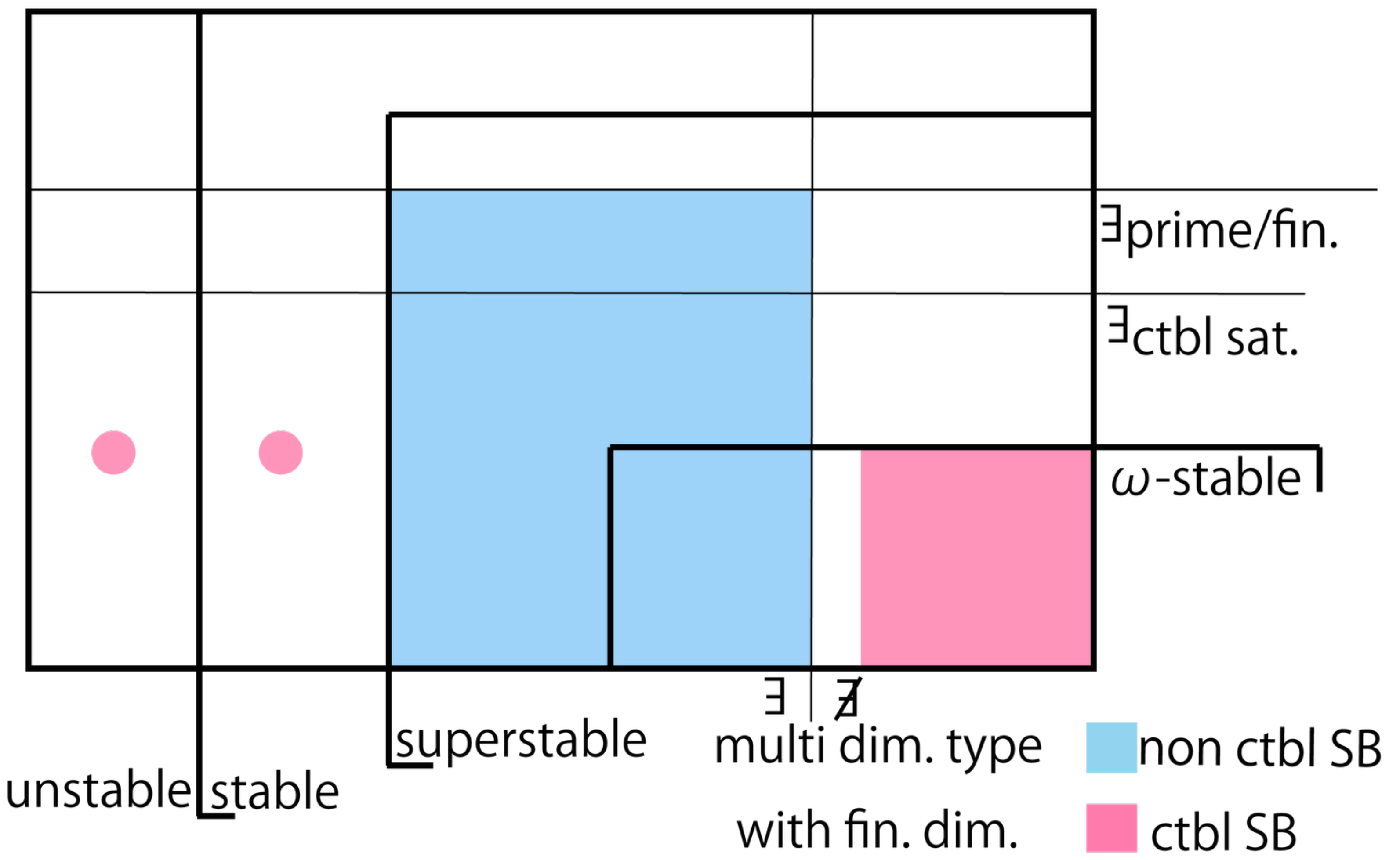
Superstable small theories with a multi dim'l type, which has fintite dim'l in some model, do not have ctbl SB.



Prop.11.(T.)

Superstable theories,which have prime model over any finite set, with a multi dim'l type ... do not have ctbl SB.

class of countable theories



Further preliminaries for prop.11.

Notation.

$p \in S(a), a \equiv b \to p_b \in S(b)$ means copy of p.

Def.12.

p is a multi dimensional type

if $p \perp \emptyset$.

Precious statement of prop.11.

Prop.11.

Let T be a superstable countable theory which has prime models over each finite set.

Suppose T has a multi dimensional type with finite dimension for some model of T.

Then T does not have ctbl SB.

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we need two models of T which elementary bi-embeddable and non isomorphic.

Outline of Proof :

We construct two models by back and forth argument and downward Lowenheim-Skolem argument.

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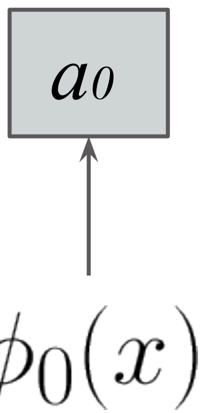
M0:



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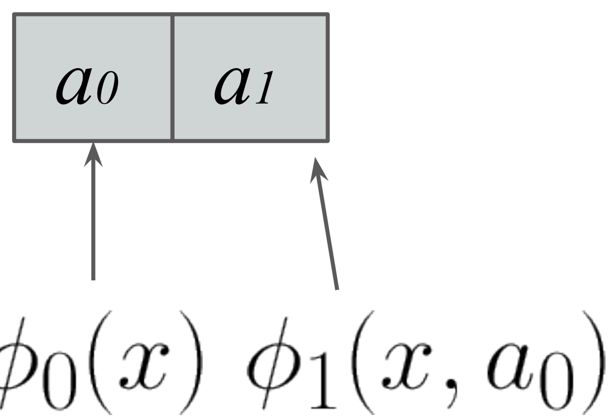
M_1:



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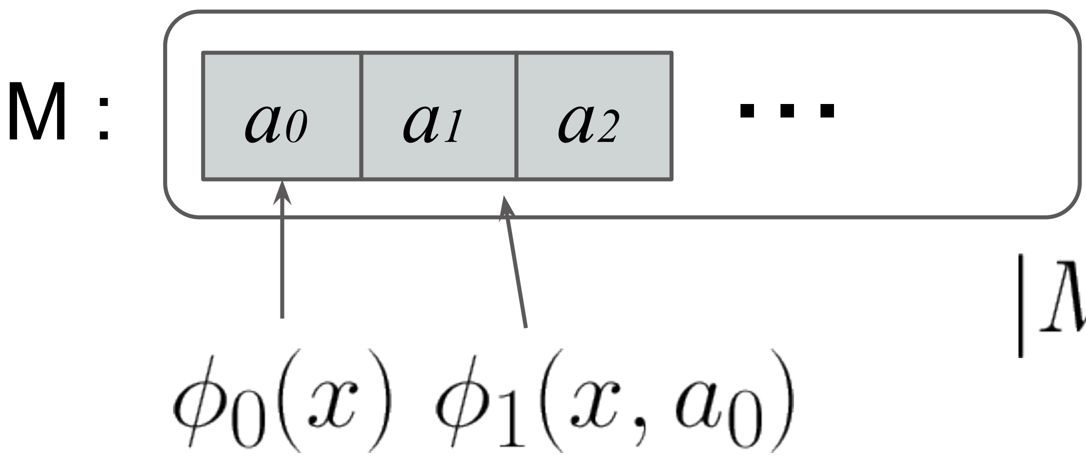

$$|M| = \omega, M \models T$$

Outline of Proof :

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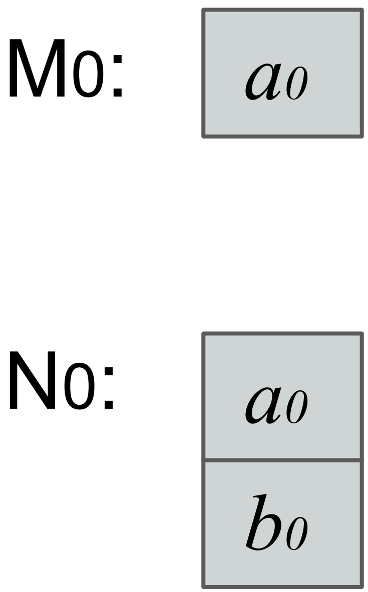

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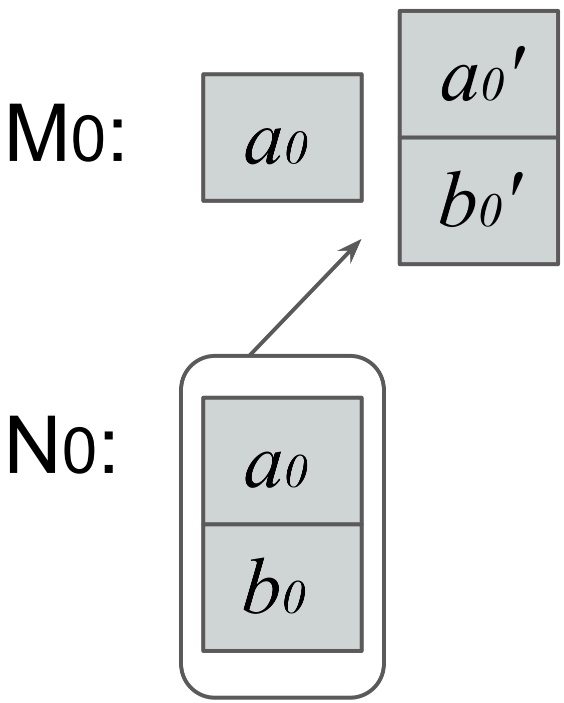

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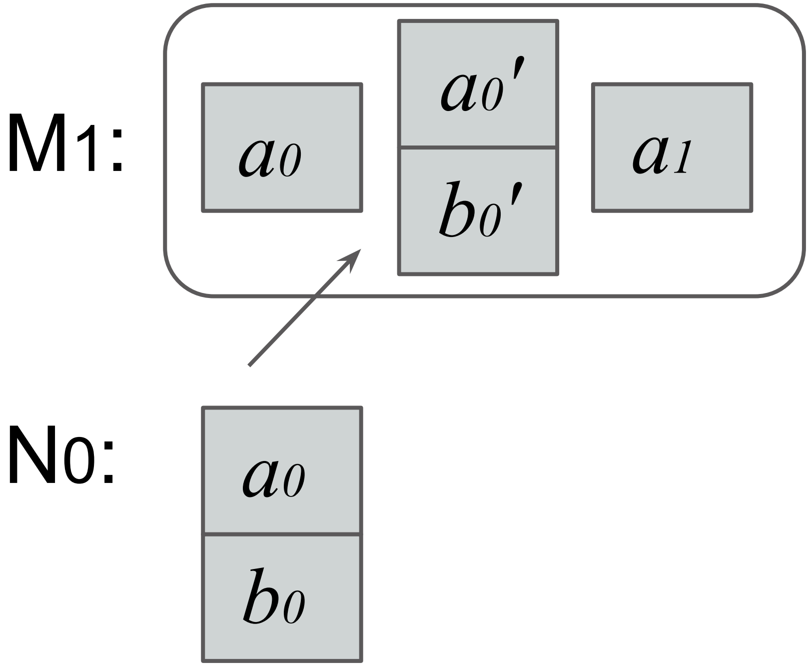

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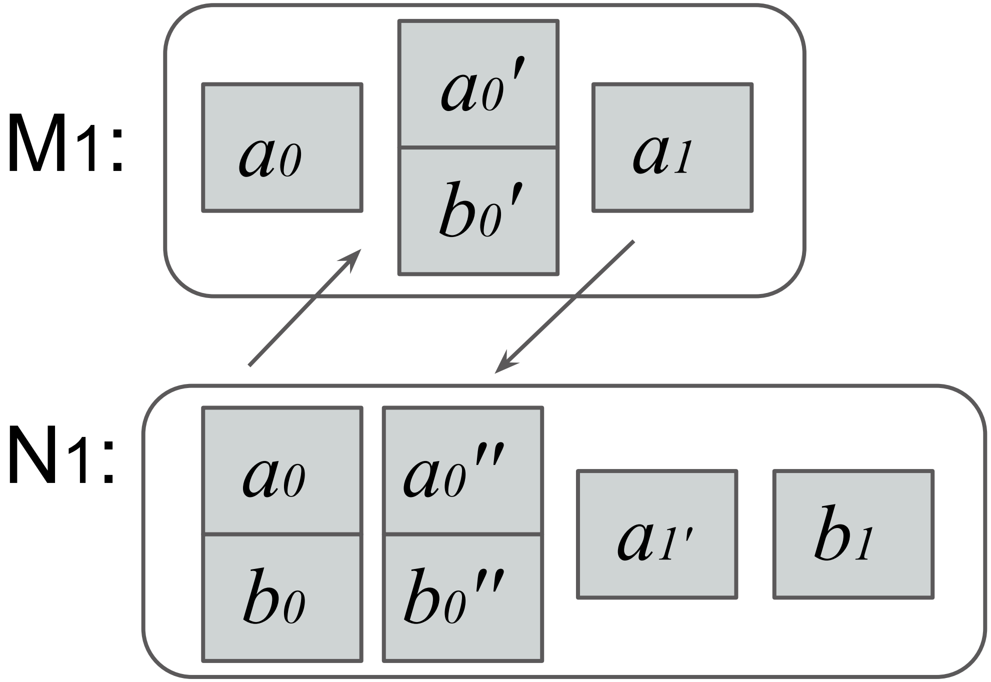

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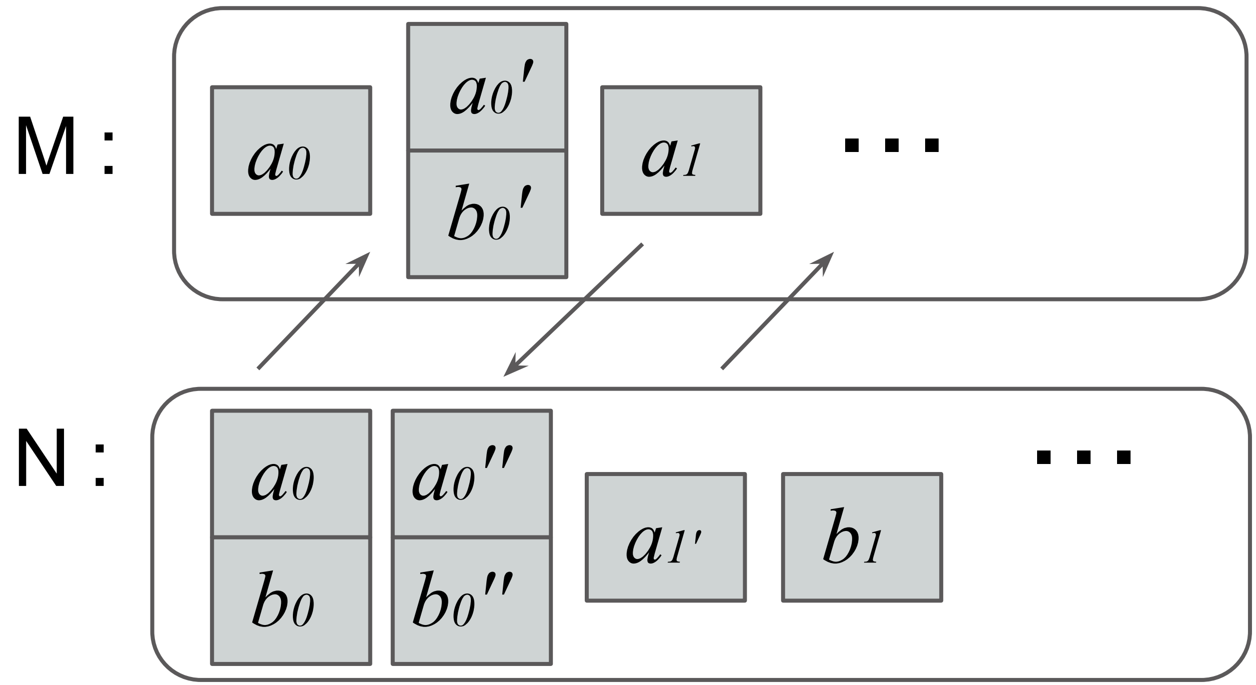

$$M \prec N,$$
$$N \prec M$$

Outline of Proof :

Let p in $S(a)$ be a multi dimensional regular type with finite dimension in some model.

To make a gap of models M, N, in each step,

1. we increase dimension in M of p_b for each b in M,which equivalent to a ;
2. we preserve dimension in N of p.

Then dim(M,p_b) = ω and dim(N,p)< ω.

$$M \not\cong N$$

Proof of prop.11.

Let p in $S(a)$ be a multi dimensional regular type with finite dimension in some model.

We will define tuples $< M_l, N_l, f_l >_{l \in \omega}$.

M_l, N_l are finite fragments of models.

$f_l : N_{l-1} \to M_l$ is an elementary map.

(0). $N_{l-1} = M_0 = \varnothing, N_0 = \{a\}$

$M_l \subset M_{l+1}, \ldots, f_l \subset f_{l+1}$

Proof of prop.11.

Induction hypothesis of $< M_l, N_l, f_l >_{l\in\omega}$:

(1). $a \downarrow M_l, N_l = aM_l\bar{c}$

where $tp(\bar{c}/M_l a)$ is isolated.

(2).there exists a realization of $p_b|M_l$ in M_{l+1} for each $b \in M_l$ with $a \equiv b$.

Proof of prop.11.

・Construction of M_{l+1} :

Let M_{l+1} be $M_l\bar{d}\bar{e}$ with witness of some fml, where

(1-1). $\bar{d}$ is a tuple of realizations of $\{p_b : M_l \in b, a \equiv b\}$ satisfying $\bar{d} \underset{M_l}{\downarrow} a$.

(1-2). $\bar{e}$ is a tuple satisfying $\bar{e} \underset{M_l\bar{d}}{\downarrow} a$ and

$$tp(\bar{e}, f_l(N_{l-1})) = tp(N_l - N_{l-1}, N_{l-1}).$$

Proof of prop.11.

・Checking ind. hyp. for M_{l+1} :

By (1-1), ind. hyp. of (2) holds.

By (1-1),(1-2) and taking wittness of a fml to be independently, we get $a \downarrow M_{l+1}$.

・Construction of f_{l+1} :

Let f_{l+1} be $f_l \cup < N_l - N_{l-1}, \bar{e} >$.

Proof of prop.11.

・Construction of N_{l+1} :

By ind. hyp. of (2), $N_l = aM_l\bar{c}$.

By fixing construction of M_{l+1} , we may assume that $tp(\bar{c}/aM_{l+1})$ is isolated.

We can take a realization c' of some fml such that $tp(\bar{c}c'/aM_{l+1})$ is isolated.

So let N_{l+1} be $aM_{l+1}\bar{c}c'$.

Cleary, ind.hyp. (2) holds.

Proof of prop.11.

$$M = \cup_l M_l, N = \cup_l N_l, f = \cup_l f_l$$

Then M,N are countable models of T which are elementary bi-embeddable.

Clearly $dim(M, p_b) = \omega$ for each $b \in M s.t. a \equiv b$

Proof of prop.11.

We will show $dim(N,p) < \omega$.

We may assume $dim(M[a],p) = 0$ where $M[a]$ is a prime model over a.

Suppose $\exists i \in N, i \models p$. So there is l s.t. $i \in N_l$.
Since $N_l = aM_l\bar{c}$, $tp(i/aM_l)$ is isolated.
Then $i \not\downarrow_a M_l$ because $dim(M[a],p) = 0$ and the open mapping theorem.

Proof of prop.11.

On the other hand, $i \underset{a}{\downarrow} M_l$ follows from $p \perp \emptyset$ and $a \downarrow M_l$.

This is a contradiction. So $\nexists i \in N, i \vDash p$ i.e. $dim(N,p) = 0$.

Then $M \not\cong N$.

References

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