数論セミナーのお知らせ

今回は三人の方にお話いただきます。

日時: 2024 年 2 月 19 日 (月)
場所: D814

14:00- William Mance (Adam Mickiewicz University)

Title: Borel complexity of sets of normal numbers via generic points in subshifts with specification

Abstract: We study the Borel complexity of sets of normal numbers in several numeration systems. Taking a dynamical point of view, we offer a unified treatment for continued fraction expansions and base r expansions, and their various generalisations: generalised Lüroth series expansions and β-expansions. In fact, we consider subshifts over a countable alphabet generated by all possible expansions of numbers in $[0, 1)$. Then normal numbers correspond to generic points of shift-invariant measures. It turns out that for these subshifts the set of generic points for a shift-invariant probability measure is precisely at the third level of the Borel hierarchy (it is a Π^0_3-complete set, meaning that it is a countable intersection of F_σ-sets, but it is not possible to write it as a countable union of G_δ-sets). We also solve a problem of Sharkovsky–Sivak on the Borel complexity of the basin of statistical attraction. The crucial dynamical feature we need is a feeble form of specification. All expansions named above generate subshifts with this property. Hence the sets of normal numbers under consideration are Π^0_3-complete.

15:15- Xiang Gao (Hubei University)

Title: On an Erdős similarity problem in the large

Abstract: In a recent paper, Kolountzakis and Papageorgiou ask if for every $\rho \in (0, 1)$, there exists a set $S \subset R$ such that $|S \cap I| \geq 1 - \rho$ for every interval $I \subset R$ with unit length, but that does not contain any affine copy of a given increasing sequence of exponential growth or faster. This question is an analogue of the well-known Erdős similarity problem. In this talk, we show that for each sequence of real numbers whose integer parts form a set of positive upper Banach density, one can explicitly construct such a set S that contains no affine copy of that sequence. Since there exist sequences of arbitrarily rapid growth that satisfy this condition, our result answers Kolountzakis and Papageorgiou ’s question in the affirmative. A key ingredient of our proof is a generalization of results by Amice, Kahane, and Haight from metric number theory. This is joint work with Yuveshen Mooroogen and Chi Hoi Yip.

16:30- Tetsuro Kamae (Osaka Metropolitan Univ)

Title: Some word problems

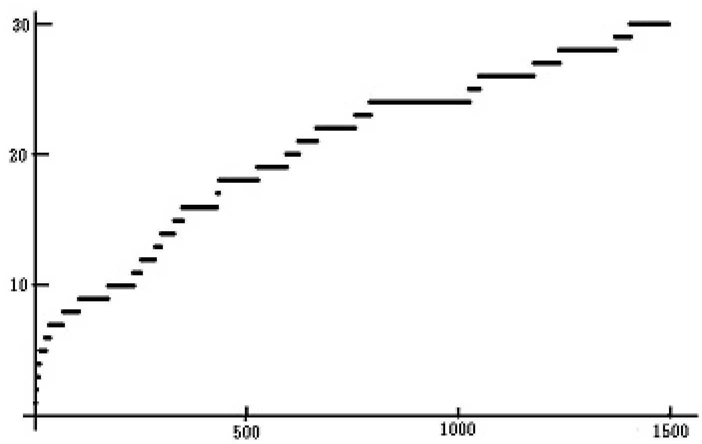
Abstract: Prof. Shigeki Akiyama (Tsukuba) asked me the following problem:

(*) Given a nondecreasing sequence of integers $\{a_n;\ n \in \mathbb{N}\}$ ($\mathbb{N} = \{0, 1, 2, \cdots\}$) having bounded gaps $a_{n+1} - a_n$ ($n \in \mathbb{N}$). Does there exists $\{i < j < k\} \subset \mathbb{N}$ such that $2j = i + k$ and $2a_j = a_i + a_k$?

It was originally posed by Pirillo and Varricchio in 1994 and seems to be still open. I cannot answer to this problem, but I introduce here some related word problems and experimental result, one suggesting "no" and the other suggestion "yes" to this problem.

Let $\mathbb{A}$ be a nonempty finite set of symbols. For a finite word $\xi = \xi_1\xi_2 \cdots \xi_l$ over $\mathbb{A}$ and $a \in \mathbb{A}$, denote $|\xi|_a = \#\{i;\ \xi_i = a\}$. Then, the *abelinization* of ξ is defined to be the vector $(|\xi|_a;\ a \in \mathbb{A})$. Take any $\mathbb{A}$ with $\#\mathbb{A} = \#\{a_{n+1} - a_n;\ n \in \mathbb{N}\}$ and a bijection $\tau : \{a_{n+1} - a_n;\ n\} \to \mathbb{A}$. We define $\alpha \in \mathbb{A}^{\mathbb{N}}$ so that $\alpha(n) = \tau(a_{n+1} - a_n)$ ($\forall n \in \mathbb{N}$).

We say that α is *repetition free* if $\xi\xi$ is not a factor of α for any nonempty finite word ξ. We also say that α is *abelian repetition free* if $\xi\eta$ is not a factor of α for any nonempty finite words ξ, η having the same abelinization. It holds that if α has an abelian repetition, then the answer to (*) is "yes". In the case $\#\mathbb{A} = 3$, it is well known that there exists a repetition free $\alpha \in \mathbb{A}^{\mathbb{N}}$ but there does not exist an abelian repetition free $\alpha \in \mathbb{A}^{\mathbb{N}}$. Hence, if $\#\{a_{n+1} - a_n;\ n \in \mathbb{N}\} \leq 3$, then the answer to (*) is "yes". In the case $\#\mathbb{A} = 4$, it is known that there exists an abelian repetition free $\alpha \in \mathbb{A}^{\mathbb{N}}$ (V. Keranen). This suggests that the answer to (*) in general is "no". In fact, there is no 2-dimensional arithmetic progression of length 4 among $\{(n, a_n);\ n \in \mathbb{N}\}$ if $\{a_{n+1} - a_n;\ n \in \mathbb{N}\} = \{0, 1, 3, 4\}$ (J. Cassaigne et al.), whereas (*) is asking about length 3.



On the other hand, starting with $a_0 = a_1 = 0$, define a_n ($n = 2, 3, \cdots$) inductively. Assume that a_i ($i = 0, 1, \cdots, n-1$) are already defined. Define a_n to be the minimum integer $a \geq a_{n-1}$ such that there is no $\{i < j < n\} \subset$

$\{0, 1, \cdots, n\}$ with $2j = i + n$ and $2a_j = a_i + a$. The sequence $(a_{n+1} - a_n;\ n \in \mathbb{N})$ thus obtained is:

$$0102010310141020102301061201040107131210432610314201023010342106351525051210172 0 \cdots$$

The above graph is the value $\#\{a_i - a_{i-1};\ i = 1, 2, \cdots, n\}$ defined like this as a function of n. It looks like this function increases unboundedly so that as long as $a_{n+1} - a_n$ $(n \in \mathbb{N})$ stays bounded, the relation that $2j = i + k$ and $2a_j = a_i + a_k$ for some $i < j < k$ is inevitable and the answer to (*) is “yes”.

Acknowledgment: I appreciate Prof. Luca Zamboni (Lyon) for giving me important informations about repetition of words.

References:

Veikko Keranen, Abelian squares are avoidable on 4 letters, Lecture Note in Computer Science, July 1992

Julien Cassaigne, James D. Currie, Luke Schaeffer and Jeffrey Shallit, Avoiding Three consecutive blocks of the same size and same sum, arXiv:1106.5204v3[cs.DM], Aug. 2011

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