

High-Dimensional Precision Matrix Quadratic Forms: Estimation Framework for $p > n$

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We propose a novel estimation framework for quadratic functionals of precision matrices in high-dimensional settings, particularly in regimes where the feature dimension p exceeds the sample size n . Traditional moment-based estimators with bias correction remain consistent when $p < n$, i.e., when p/n tends to a constant less than one. However, they break down entirely once $p > n$, highlighting a fundamental distinction between the two regimes due to rank deficiency and high-dimensional complexity. Our approach resolves these issues by combining a spectral-moment representation with constrained optimization, resulting in consistent estimation under mild moment conditions.

The proposed framework provides a unified approach for inference on a broad class of high-dimensional statistical measures. We illustrate its utility through two representative examples: the optimal Sharpe ratio in portfolio optimization and the multiple correlation coefficient in regression analysis. Simulation studies demonstrate that the proposed estimator effectively overcomes the fundamental $p > n$ barrier where conventional methods fail.