

A note on estimation of restricted (ordered, upper (lower)-bounded) scale parameter of Gamma distribution

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Summary. First, we introduce the estimation of a linear function of two ordered scale parameters of two Gamma distributions. A necessary and sufficient condition for the ratio of two coefficients, which guarantees that the maximum likelihood estimator (MLE) and the modified MLE dominate the crude unbiased estimator (UE) and the admissible estimator based solely on one sample under quadratic loss are derived, respectively. (Chang and Shinozaki (2002,2008)). Next, an admissible estimator of the upper-bounded scale parameter of a Gamma distribution is derived and the simultaneous estimation of p upper-bounded scale parameters of Gamma distributions are considered. Moreover, simultaneous estimation of two ordered scale parameters of two Gamma distributions is investigated in terms of the mean square error (MSE).

The Gamma distribution plays an important role in reliability theory, as it serves often as a model for lifetime times and waiting times, leading to the necessity of estimating the parameters of a Gamma distribution. Usually, the estimation problem is solved assuming that there are no restrictions with respect to the values of the parameter. However, in reality the parameters are always restricted and very often prior information about the restriction is available. In this case, the usual methods of estimation turn out to be inappropriate. Therefore, we consider for the scale parameter some types of constraints, for example, bounded above or order restrictions between two scale parameters, and derive estimators, which take into account the restrictions. The suggested estimators perform better than the usual estimators.

Problems concerning statistical inference for restricted parameters are of considerable interest. The earlier works related to statistical inference under restrictions are reviewed by Barlow et al (1972), Robertson and Wright (1988). The more recent literature about restricted statistical inference after 1988 is reviewed in two monographs, Silvapulle and Sen (2004), Eeden (2006). Many authors have considered inference on parameters in case of a is simple order restriction or tree order restriction on parameters and have suggested estimators, which improve the performance of unbiased estimators, especially for the normal distribution.

Katz (1961) has shown the admissibility or minimaxity of estimators when an upper or lower bound restriction is given in the case of the expectation parameter in exponential family under squared error loss. Casella and Strawderman (1981) proposed a minimax estimator for a normal mean that is restricted to a bounded interval. Berry (1993) considered admissible minimax estimators of the location parameter for the exponential distribution, when upper bound, lower bound or two-sided restrictions are given. Eeden (1995) suggested an admissible estimator for the scale parameter of a Gamma distribution with given lower bound under scale invariant squared error loss. When there is an order restriction between two scale parameters of two Gamma distributions, Chang and Shinozaki (2002,2008) have considered the estimation of a linear function of two ordered scale parameters of two Gamma distributions. They derived a necessary and sufficient condition for the ratio of two coefficients, which guarantees that the maximum likelihood estimator (MLE) and the modified MLE dominate the crude unbiased estimator (UE) and the admissible estimator based solely on one sample under quadratic loss, respectively. Marchand and Strawderman (2005) improved the minimum risk equivariant estimator of a scale parameter under a lower bound restriction.

Although Katz(1961) has given a proof of the admissibility for an estimator of the expectation parameter in the exponential family, when there is a lower bound(upper bound) restriction, Eeden(1995) has pointed out that there is an error in Katz's proof and showed that Katz's condition is not satisfied for the Gamma scale parameter with lower bound restriction.

On the other hand, Berger(1980) developed a general technique for simultaneous estimation of Gamma scale parameters to improve the standard estimators under several types of loss functions that include the sum of squared error loss and scale invariant squared error loss.

In Section 1, we first introduce the estimation of a linear function of two ordered scale parameters of two Gamma distributions (Chang and Shinozaki (2002, 2008)) in term of MSE and entropy loss. In Section 2, we discuss the admissible estimation of an scale parameter, which is bounded above, of the Gamma distribution under squared error loss. In Section 3, we consider the simultaneous estimation of Gamma scale parameters with upper bound and lower bound restrictions under sum of squared error loss and sum of scale invariant squared error loss. In Section 4, we consider the simultaneous estimation of two ordered scale parameters of two Gamma distributions.