

A nonparametric test of spherical symmetry applicable to high dimensional data

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Abstract

We develop a test for spherical symmetry of a multivariate distribution P that works well even when the dimension of the data d is larger than the sample size n . We propose a non-negative measure of spherical asymmetry $\zeta(P)$ such that $\zeta(P) = 0$ if and only if P is spherically symmetric. We construct a consistent estimator of $\zeta(P)$ using the data augmentation method and investigate its large sample properties. The proposed test based on this estimator is calibrated using a novel resampling algorithm. Our test controls the type I error, and it is consistent against general alternatives. We also study its behavior for a sequence of alternatives $(1 - \delta_n)F + \delta_n G$, where $\zeta(G) = 0$ but $\zeta(F) > 0$, and $\delta_n \in [0, 1]$. When $\limsup \delta_n < 1$, for any G , the power of our test converges to unity as n increases. However, if $\limsup \delta_n = 1$, the asymptotic power of our test depends on $\lim n(1 - \delta_n)^2$. We establish this by proving the minimax rate optimality of our test over a suitable class of alternatives and showing that it is Pitman efficient when $\lim n(1 - \delta_n)^2 > 0$. Moreover, our test is provably consistent for high-dimensional data even when d grows with n . When the center of symmetry is unspecified, naive centering may lead to size distortions. We therefore propose a modified procedure based on pairwise differences that retains the desired level and other theoretical properties of the test. Simulation studies illustrate the theoretical findings and demonstrate favorable performance relative to existing methods.

Keywords: Consistency; Contiguous alternatives; Data augmentation; Minimax rate optimality; Pitman efficiency; Spherical symmetry.