Diophantine Analysis and Related Fields 2024
Scheduled Talks and Their Abstracts

Monday 4th March

13:30–14:20 **Shinichi Yasutomi** (Toho University)
Title: **Simultaneous convergent continued fraction algorithm with two places**
Abstract: Let p be a prime number and K be a field with embeddings into $\mathbb{R}$ and $\mathbb{Q}_p$. We propose some algorithms that generate continued fraction expansions converging in $\mathbb{Q}_p$ and are expected to simultaneously converge in both $\mathbb{R}$ and $\mathbb{Q}_p$. This algorithm produces finite continued fraction expansions for rational numbers. In the case of $p = 2$ and if K is a quadratic field, the continued fraction expansions generated by this algorithm converge in $\mathbb{R}$, and they are eventually periodic or finite. For an element α in K, let p_n/q_n denote the n-th convergent. There exist constants u_1 and u_2 in $\mathbb{R}_{>0}$ with $u_1 + u_2 = 2$, and constants C_1 and C_2 in $\mathbb{R}_{>0}$ such that $|\alpha - p_n/q_n| < C_1/|q_n|^{u_1}$ and $|\alpha - p_n/q_n|_2 < C_2/|q_n|^{u_2}$. Here, $|\cdot|_2$ represents the 2-adic distance. For prime numbers $p > 2$, we present numerical experiences and some conjectures.

14:30–15:20 **Yu Yasufuku** (Nihon University)
Title: **Bounding GCD's of evaluations of two trivariate Polynomials**
Abstract: Ru and Vojta devised an efficient way of applying Schmidt subspace theorem, via an algebraic-geometric invariant. This has already been applied to analyze GCD's in various settings, but some subtle difficulties arise for example when we try to analyze the GCD of $f(a, b, c)$ and $g(a, b, c)$, where f and g are integer-coefficient polynomials of degrees 2 and 1 respectively and a, b, c are integers. We discuss the algebro-geometric difficulties in such cases, and compare what we can obtain from Ru–Vojta method with what is predicted from Vojta's conjecture.

15:40–16:20 **Yuta Suzuki** (Rikkyo University)
Title: **Some remarks on counting relatively prime pairs**
Abstract: It is a well-known result that the density of relatively prime pairs with respect to the set of pairs of integers is $6/\pi^2$. In this talk, we consider variations of this result with the relatively prime pairs from a given sequence. A problem of the classical method is that it stops working if the counting function of the given sequence is of the size $\ll x^{1/2}$. In this talk, we remark that we can overcome this difficulty by a simple trick. As applications, we apply this trick to the $[x/n]$-sequence, a conjecture of Banks–Shparlinski on palindromic numbers and the Piatetski-Shapiro sequences. This talk is based on joint work with Kota Saito, Wataru Takeda and Yuuya Yoshida and joint work with Hirotaka Kobayashi and Ryota Umezawa.

16:30–17:10 **Wataru Takeda** (Tokyo Universitu of Science)
Title: **Multiplicative Titchmarsh divisor problem for the [x/n]-sequence**
Abstract: The Titchmarsh divisor problem, a variant of the ordinal divisor problem, is the problem of giving the asymptotic behavior of the summatory function of the number of divisors of shifted primes. In this talk, I talk about certain "multiplicative" analogs of the Titchmarsh divisor problem. We consider two types of analogs

and derive asymptotic formulas for such multiplicative Titchmarsh divisor problems for some small arithmetic functions, and the Euler totient function with the von Mangoldt function. This work is a joint-work with Kota Saito, Yuta Suzuki, and Yuuya Yoshida.

17:20–18:00 **Tomohiro Yamada** (Osaka University)
Title: **On Gauss-Kraïtchik formula for cyclotomic polynomials via symmetric function**
Abstract: For any odd integer $d \geq 3$, the d-th cyclotomic polynomial $\Phi_d(X)$ can be represented in the way $4\Phi_d(X) = \Psi_d(X)^2 - D\Xi_d(X)^2$ with $D = (-1)^{(d-1)/2}d$ using two polynomials $\Psi_d(X) = \sum_{n=0}^{d'} a_{d,n}X^{d'-n}$ and $\Xi_d(X) = \sum_{n=1}^{d'} b_{d,n}X^{d'-n}$ of integer coefficients, which is proved by Gauss for prime cases and by Kraïtchik for general cases. We obtain upper bounds for absolute values of these coefficients. If $d \geq 3$ is odd and squarefree, then $|a_{d,n}| + |b_{d,n}|\sqrt{d} < 2\binom{G_{d,n}+n-1}{n}$, where $G_{d,t} = \max\{\varphi(f)/2 : f \mid d, 1 < f \leq t\} \cup \{|1+\sqrt{d}|/2\}$. As an application, we show that if $d \geq 5$ is odd and squarefree and $x > 2G_d^2$ with $G_d = G_{d,\varphi(d)/4}$, then

$$\left|\frac{\Xi_d(x)}{\Psi_d(x)} - \frac{1}{2x-\mu(d)}\right| < \frac{x}{(2x-\mu(d))\sqrt{d}}\left(\left(1-\frac{1}{x}\right)^{-G_d} - 1 - \frac{G_d}{x}\right).$$

Thuesday 5th March

9:20–10:10 **István Pink** (University of Debrecen), **Takafumi Miyazaki** (Gunma University)
Title: **Number of solutions to a special type of unit equations in two unknowns III**
Abstract: This talk is a continuation of the one given by the first named speaker at the previous DARF on March 3, 2023. The topic is the best possible general estimate on the number of solutions to a special type of unit equations in two unknowns over the rationals. It is conjectured that for any fixed relatively prime positive integers a, b and c all greater than 1 there is at most one solution to the equation $a^x + b^y = c^z$ in positive integers x, y and z, except for specific cases. In this talk we give a brief introduction to the conjecture, and present our results with their proofs, which in particular enables us to find some new values of c being presumably infinitely many such that for each such c the conjecture holds true except for only finitely many pairs of a and b. Most importantly we prove that if $c = 13$ then the equation has at most one solution, except for $(a, b) = (3, 10)$ or $(10, 3)$. Our study can also confirm a large part of Terai's conjecture involving the condition that all x, y and z are greater than 1.

10:20–11:10 **Noriko Hirata-Kohno** (Nihon University)
Title: **Diophantine approximations and O-Minimality**
Abstract: In the framework of the Pila-Wilkie theorem and O-minimality, Diophantine approximation has been recently applied by several authors. In this talk, we explain how Diophantine approximation, in particular transcendence method applies to the counting question of points of low transcendence degree, under suitable assumptions.

This is a joint research with G. Binyamini (the Weizmann Institute) and M. Kawashima (Nihon University).

11:20–12:00 **Makoto Kawashima** (Nihon University)
Title: **Gauss' continued fraction and irrationality**
Abstract: In 1813, C. F. Gauss introduced a general continued fraction that represents the ratio of two Gauss hypergeometric functions ${}_2F_1$ by their three term contiguous relations. While this continued fraction expansion is useful, explicitly determining its numerator and denominator is not widely known. However, it is known that these numerator and denominator are related to the Pade approximation of contiguous Gauss hypergeometric functions. In this lecture, we explain the explicit construction of $(n-1, n)$ and (n, n)-Pade approximations of contiguous Gauss hypergeometric functions derived from the contiguous relation of Gauss's hypergeometric function. Additionally, we introduce a sharp irrationality measure of value of the ratios of contiguous Gauss hypergeometric functions at an algebraic number as an application. This is a joint work with Akihito Ebisu.

14:00–14:50 **Yasutsugu Fujita** (Nihon University)
Title: **Bounding the size of D(±1)-tuples in imaginary quadratic rings**
Abstract: Let R be a commutative ring with unity. For a nonzero integer n, a set $\{a_1, a_2, \ldots, a_m\}$ of m distinct elements in $R \setminus \{0\}$ is called *a $D(n)$-m-tuple in R* if $a_i a_j + n$ is a square in R for any i and j with $1 \leq i < j \leq m$.

In case $R = \mathbb{Z}$ and $n \in \{\pm 1\}$, sharp bounds for m are known. More precisely, He, Togbé and Ziegler in 2019 settled the folklore conjecture asserting that there exists no $D(1)$-quintuple in $\mathbb{Z}$, while Bonciocat, Cipu and Mignotte in 2022 proved the non-existence of $D(-1)$-quadruples in $\mathbb{Z}$.

Let $\mathcal{O}$ be the ring of integers in an imaginary quadratic field K. In 2021, not sharp, but absolute bounds for m in the cases where $R = \mathcal{O}_K$ and $n \in \{\pm 1\}$ were given by Adžaga and Gupta, who proved the non-existence of $D(1)$-m-tuples in $= \mathcal{O}_K$ for $m > 42$ and of $D(-1)$-m-tuples in $= \mathcal{O}_K$ for $m > 36$, respectively.

The aim of this talk is to improve both of the bounds above for m. This work is a collaboration with Mihai Cipu.

15:00–15:50 **Taka-aki Tanaka** (Keio University)
Title: **On a certain function invariant under group action with remarkable algebraic independence properties**
Abstract: Using specific linear recurrences of positive integers, we construct functions of three complex variables which are invariant under the action of the dihedral groups. For such a function, we consider the map defined on the set of orbits of the action of the dihedral group on its domain of definition and the restriction of the map to the set of algebraic points. Then we show that the restriction of the map takes algebraically independent values at any distinct orbits.

16:10–16:50 **Shintaro Murakami** (Hirosaki University)
Title: **Linear independence of certain infinite series with monomial orders**
Abstract: P. Erdős (1957) proved that for any integers $i \geq 1$ and $b, j \geq 2$ the number $\alpha_{i,j} := \sum_{n=1}^{\infty} b^{-in^j}$ is transcendental or an algebraic number with degree at least j; however, not much is known about algebraic relations among the numbers $\alpha_{i,j}$ ($i = 1, 2, \ldots, j = 2, 3, \ldots$). Recently, V. Kumar (2019) gave linear independence results

for the numbers $\alpha_{i,j}$ under some conditions, and shortly thereafter, the speaker and Y. Tachiya (2023) have removed the conditions and proved that for any integer b ($|b| \geq 2$) the numbers 1, $\alpha_{i,j}$ ($i = 1, 2, \ldots, j = 2, 3, \ldots$) are linearly independent over $\mathbb{Q}$.

In this talk, we give linear independence results for Lambert series and the above numbers $\alpha_{i,j}$. More precisely, we show linear independence over $\mathbb{Q}$ of the numbers

$$1, \qquad \sum_{n=1}^{\infty} \frac{a_{i,j}(n)}{b^{in^j}} \quad (i = 1, 2, \ldots, j = 2, 3, \ldots), \qquad \sum_{n=1}^{\infty} \frac{1}{b^{kn^\ell} - 1} \quad (k, \ell = 1, 2, \ldots),$$

where $\{a_{i,j}(n)\}_{n \geq 1}$ are sequences of integers with suitable non-zero and growth conditions. This is a joint work with Yohei Tachiya (Hirosaki University).

17:00–17:40 **Motoya Teranishi** (Kyoto University)
Title: **On potential density of integral points on the complement of some subvarieties in the projective space**
Abstract: In this talk, we will introduce our density results for integral points on the complement of a closed subvariety in the n-dimensional projective space over a number field. For instance, we consider a subvariety whose components consist of n-1 hyperplanes plus one quadric hyperpsurface in general position, or four planes in general position plus a finite number of concurrent straight lines in the 3-dimensional projective space. In these cases, under some conditions on their intersection, we show that the integral points on the complements are potentially dense.

Wedsday 6th March

9:20–10:10 **Takumi Noda** (Nihon University)
Title: **On the generating Dirichlet series of the Riemann zeta-function**
Abstract: Let $\zeta(s)$ be the Riemann zeta-function and $\zeta(s, x)$ be the Hurwitz zeta-function. It is well known that $\zeta(s, x)$ is the binomial type generating function of $\zeta(s)$:

$$\zeta(s, 1 + x) = \sum_{m=0}^{\infty} \frac{\Gamma(s + m)}{\Gamma(s)\, m!} \zeta(s + m)(-x)^m$$

for $|x| < 1$ and $s \in \mathbb{C} \setminus \{1\}$, which gives a number of formula involving the Riemann zeta-function.
In this talk, we start with the classical Goldbach's formula $\sum_{k=2}^{\infty} \{\zeta(k) - 1\} = 1$, and discuss the generating Dirichlet series of the Riemann zeta-function.

10:30–11:10 **Kota Saito** (University of Tsukuba)
Title: **The simple normality of the fractional powers of two and the Riemann zeta function**
Abstract: A real number x is called a simply normal number to base b if the base-b expansion of x has each digit $0, 1, \ldots, b-1$ appearing with average frequency tending to $1/b$. We do not determine the simple normality of many non-artificial numbers such as π, e, $\log 2$, $\sqrt{2}$, etc. In this talk, we report discovering a relation between the frequency that the digit 1 appears in the binary expansion of $2^{p/q}$ and a mean value of the Riemann zeta function on arithmetic progressions. This is joint work

with Yuya Kanado (Nagoya University).

11:20–12:00 **Hajime Kaneko** (University of Tsukuba)
Title: **Markoff and Lagrange spectra on one-sided shift spaces with cylinder orders**
Abstract: Various analogies of the Markoff-Lagrange spectrum have been investigated by many mathematicians. Multiplicative versions of the Markoff-Lagrange spectrum are related to the fractional parts of linear recurrences. Dubickas investigated the minimal limit points of certain multiplicative Markoff-Lagrange spectrum in terms of symbolic dynamical systems and certain substitution.

In this talk, we study an analogy of the Markoff-Lagrange spectrum for one-sided shift spaces with cylinder orders. As our main results, we determine the discrete parts and minimal limit points in terms of S-adic sequences, where S is an infinite set of substitutions. This is a joint work with Wolfgang Steiner.