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Title

Integral points and generators for the Mordell-Weil groups of elliptic curves

Abstract

Collaborating with Nobuhiro Terai or Tadahisa Nara, I have been studying integral points and generators for the Mordell-Weil groups $E(\mathbb{Q})$ of elliptic curves, mainly, of the form $E : y^2 = x^3 + nx$ $(n \in \mathbb{Z})$ for around ten years. In this talk, I will present some of the results and the strategies for their proofs from the research above. The following is an example of the results:

For a square-free integer m, let C_m be a curve given by $u^2 - v^4 = m$ (C_m is birationally equivalent over $\mathbb{Q}$ to $E_m : y^2 = x^3 - 4mx$).

(1) If m is expressed as $m = 3(s^4 + s^2t^2 + t^4)$ $(s, t \in \mathbb{Z},\ \gcd(s, t) = 1)$, then the two points

$$P_1 = (st + 2(s^2 + t^2), s + t), \quad P_2 = (st - 2(s^2 + t^2), s - t)$$

on the elliptic curve C_m can be extended to a basis for $C_m(\mathbb{Q})$.

(2) If $\operatorname{rank} C_m(\mathbb{Q}) = 2$, then C_m has at most eight integral points, which can be expressed as

$$(a_1, \pm b_1),\ (-a_1, \pm b_1),\ (a_2, \pm b_2),\ (-a_2, \pm b_2)$$

for some integers a_1, b_1, a_2 and b_2.