Homework

May 2024

1 Manifolds

Exercise 1.1 *Construct an canonical isomorphism* $H^2(X, \mathbb{Z}) = [X, K(\mathbb{Z}, 2)]$*, where* $K(\mathbb{Z}, 2)$ *is the Eilenberg-MacLane space.*

Here $[X, Y]$ is the homotopy set of continuous maps $f : X \to Y$

Exercise 1.2 *For a group* G *prove that any principal* G*-bundle* $P \to X$ *is isomorphic to the pull-back bundle* $f^*\pi_G$ *of* π_G *by* $f : X \to BG$*. Here* $\pi_G : EG \to BG$ *is the universal* G*-bundle.*

Exercise 1.3 *Prove* $BU(1) = K(\mathbb{Z}, 2)$ *and it is homotopy equivalent to infinite dimension complex projective space.*

Exercise 1.4 *Construct for any group* G *the classifying space* BG *and the universal principal* G*-bundle* $\pi_G : EG \to BG$*.*

Exercise 1.5 *Let* X *be a closed 4-manifold. Prove for* $\alpha, \beta \in H_2(X, \mathbb{Z})$*,* $\alpha \cdot \beta = Q(PD(\alpha), PD(\beta)) = \int_X \eta_A \wedge \eta_B$*. Here* η_A *and* η_B *are closed 2-forms corresponding to the integral classes for the natural isomorphism* $H_2(X, \mathbb{R}) \cong H^2_{dR}(X, \mathbb{R})$

Exercise 1.6 *For any element* $\alpha \in H_2(X^4)$ *is equivalent to the homology class of the zero set of a generic section* $s : X \to \mathcal{L}$ *of a line bundle* $\mathcal{L}$*.*

2 Knot theory

Exercise 2.1 *Prove any knot in* S^3 *has a Seifert surface. Use the Seifert Algorigthm.*

Exercise 2.2 *Prove that Hopf link has a Seifert surface homeomorphic to annulus.*